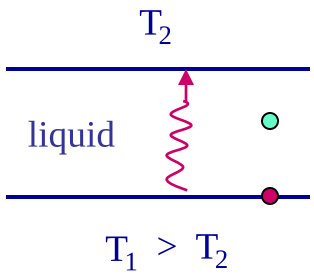
Glacial Climate Changes and Stochastic Resonance

Zoltán Rácz

Institute for Theoretical Physics
Eötvös University
E-mail: racz@general.elte.hu
Homepage: http://cgl.elte.hu/~racz

Problem: Understanding glacial-intergalcial climatic oscillations

Social aspect of the problem: Existence of witches

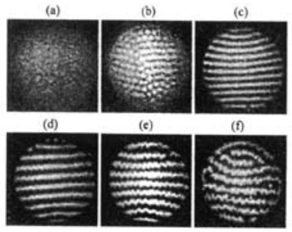


Questions: What do we know from the past (time series)?

What are the relevant features of the data?

What drives the climatic processes?

Energy- és energy-flux scales



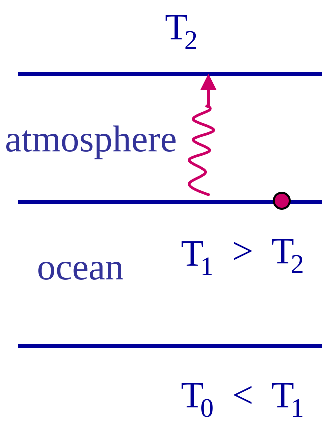
Models: External drive

Thresholds, relaxations, memory effects

Internal drive

Thresholds, feedbacks

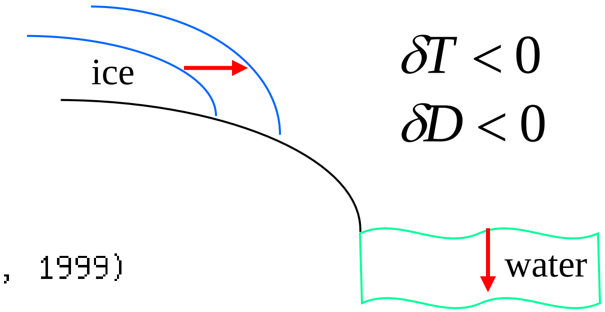
Importance of noise



Epilogue: Probability of change of opinion: Do witches exist?

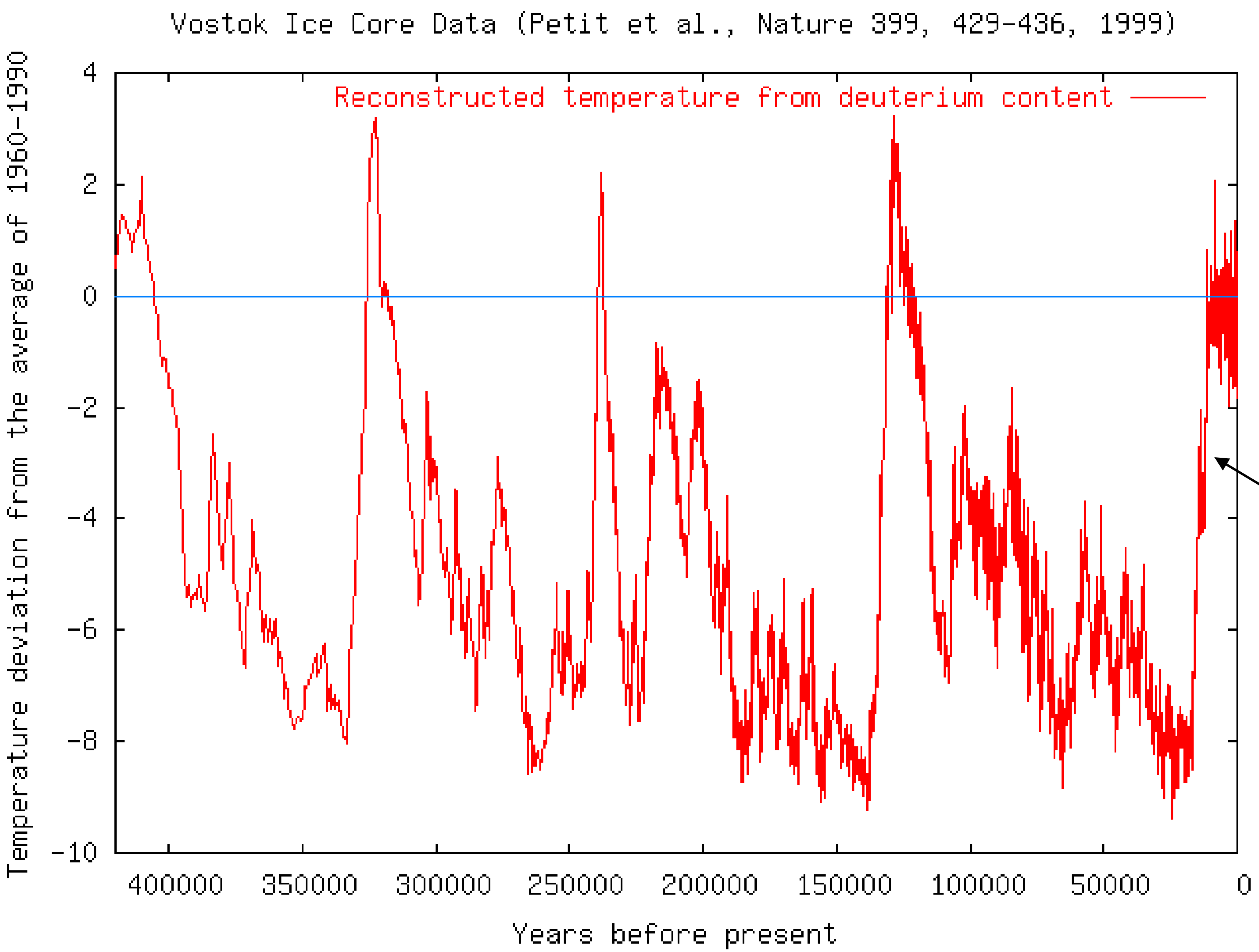
Acknow.: Z. Horváth, I. Jánosi, T. Tél and I. Daruka (Eötvös University)

The last 430 thousand years



$\delta T < 0$

$\delta D < 0$



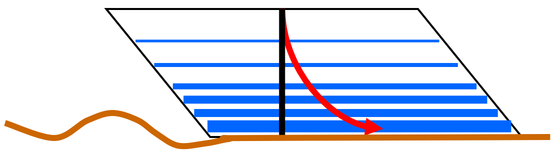
Slow cooling

Fast increase (starting from low T-s!)

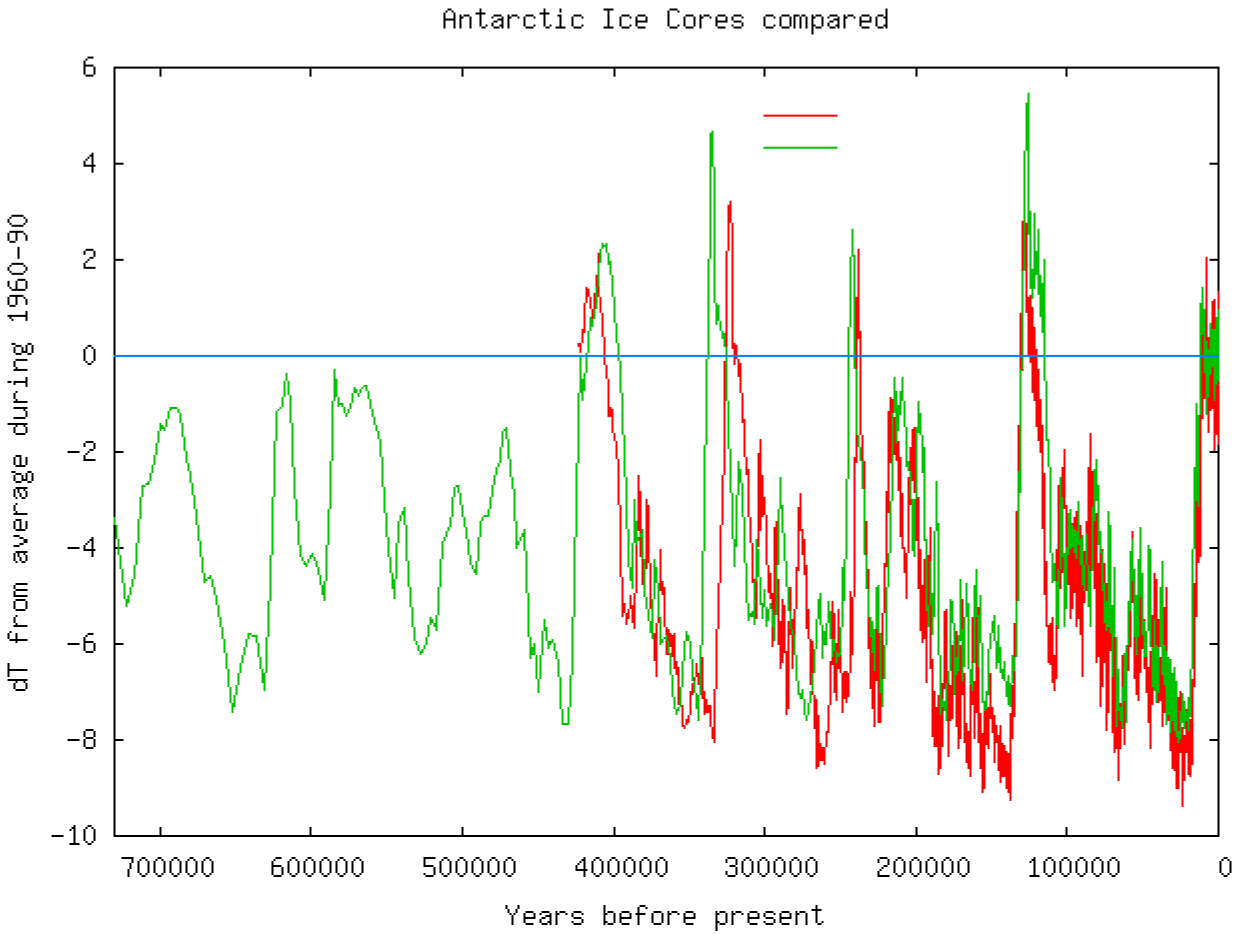
$\Delta T > 6^o C / 50 y$

Last 725 thousand years

Accuracy of data:

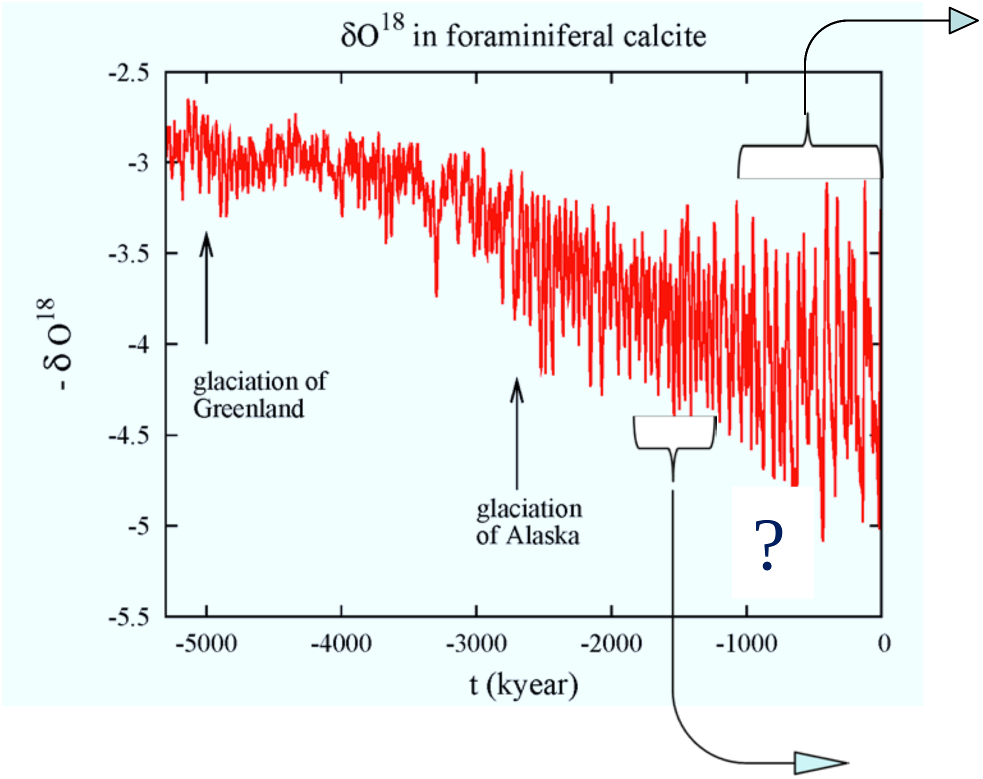


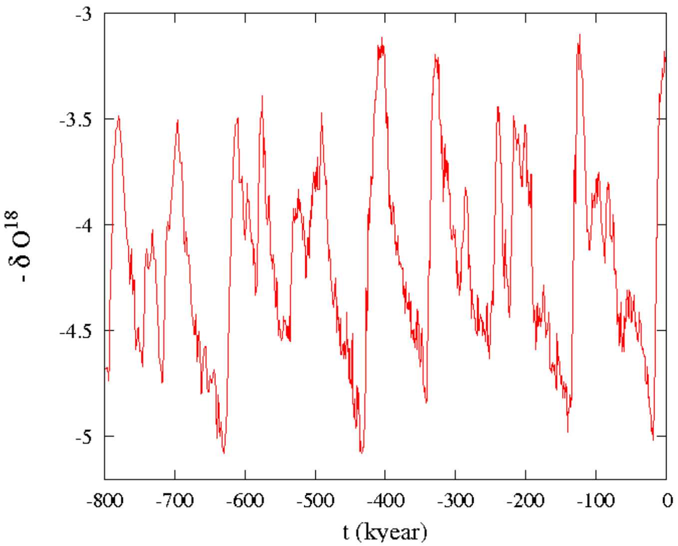
pressure
shear
roughness

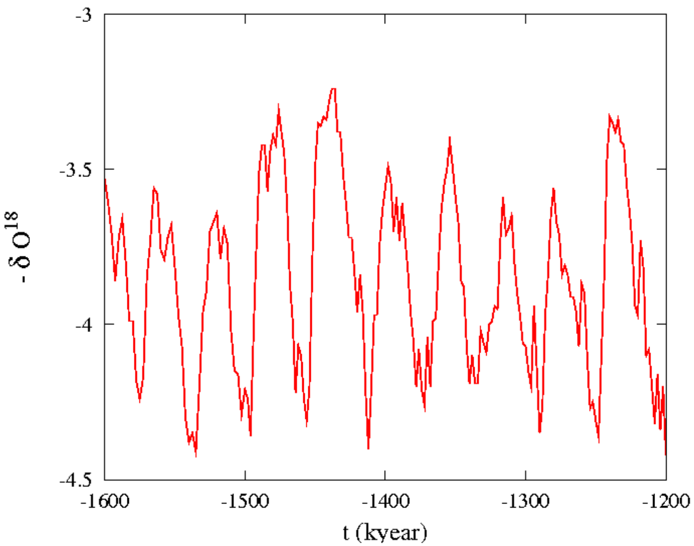


Last 5 million years

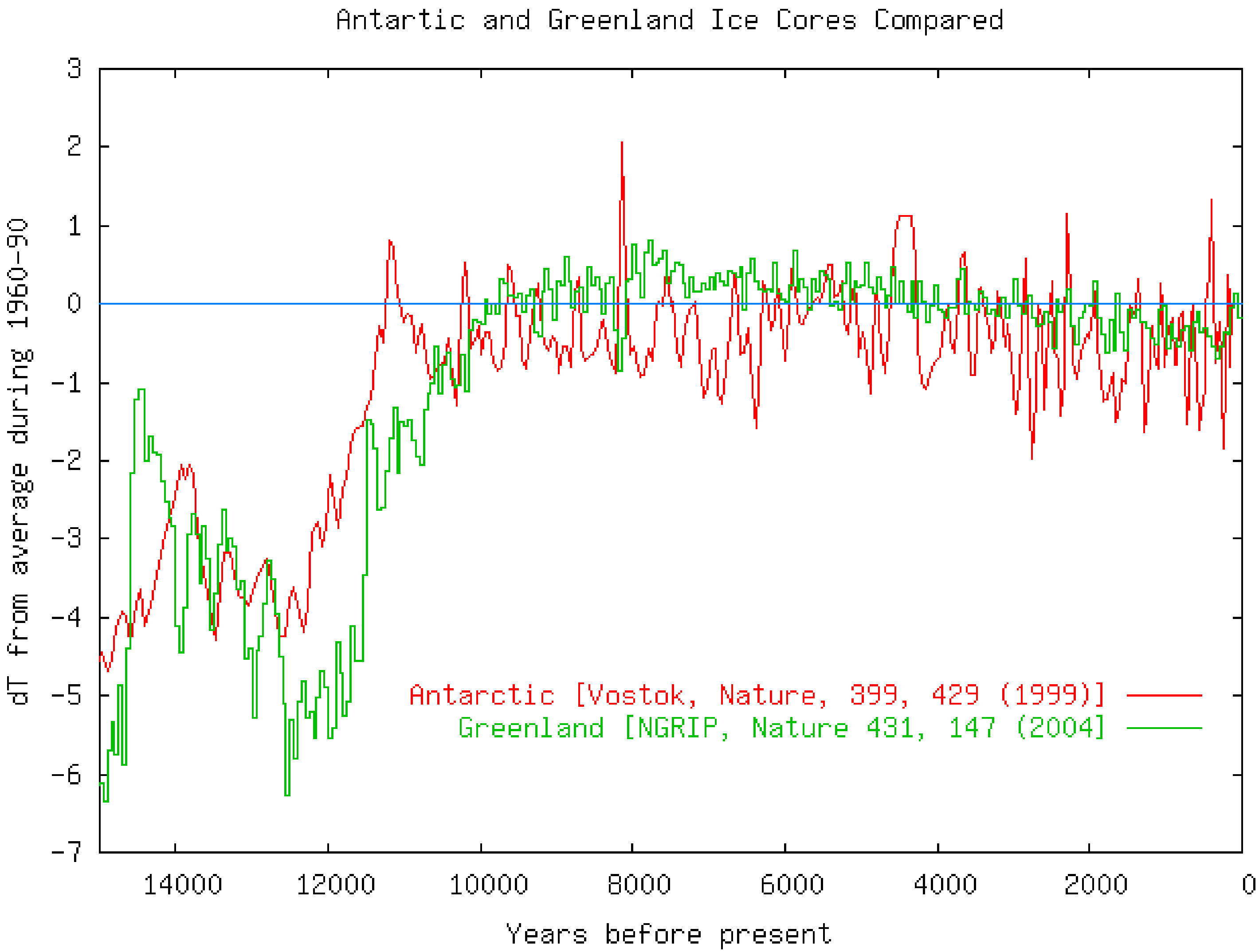
M.E. Raymo and K. Nisancioglu,
Paleoceonography, **20**, PA1003 (2003)



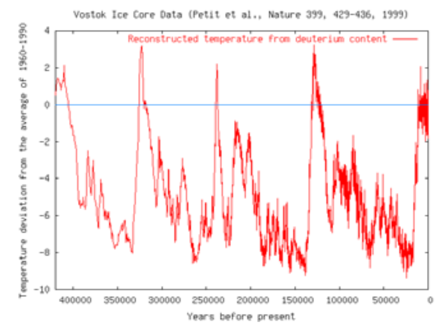




Last 15 000 years: Differences between north and south



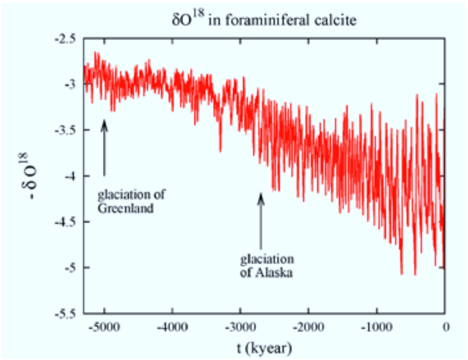
Features we would like to understand



t > -800 ky:

- strong ~100 ky period
- weaker ~41 ky period
- Directionality

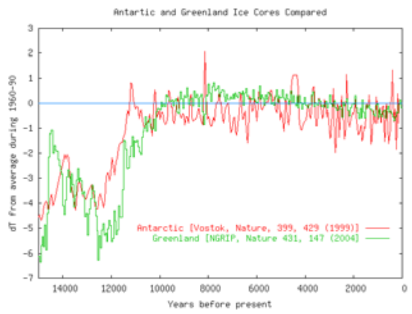
  Saw-tooth: Slow cooling, fast warming



t < -800 ky

- ~100 ky period disappears
- ~41 ky period dominating

- North and south are ~synchronized



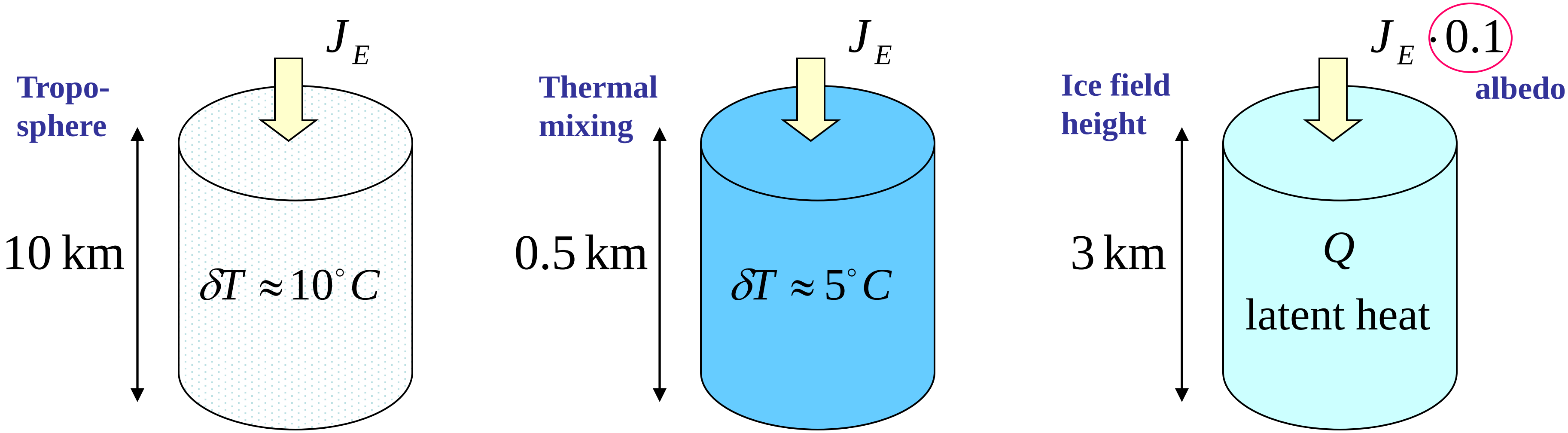
- Fluctuation spectrum is continuous

  $S(\omega) \sim \omega^{-1.8} \sim \omega^{-2.2}$

Energies and energy fluxes: Characteristic times

relaxation time of the perturbation → $$\tau \approx \frac{\delta E}{J_E}$$ ← energy perturbation; ← incoming energy flux $\approx 342.5\ \mathrm{w/m^2}$

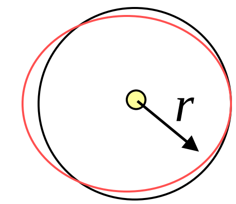
Atmosphere	Ocean	Ice fields

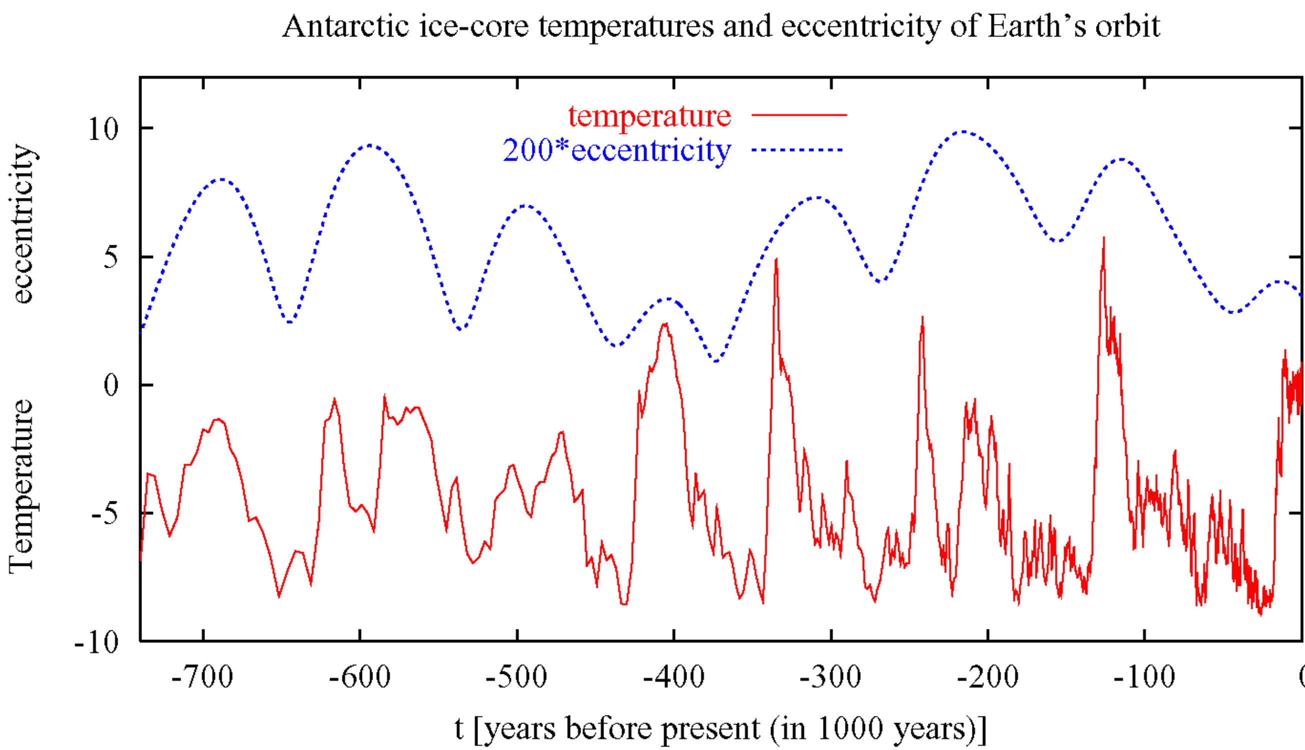


Atmosphere	Ocean	Ice fields
$\delta E \approx 2 \cdot 10^8\ J/m^2$	$\delta E \approx 2 \cdot 10^{10}\ J/m^2$	$\delta E \approx 10^{12}\ J/m^2$
$\tau \approx 5$ days	$\tau \approx 1$ year	$\tau \approx 10^2 - 10^3$ y
Weather fronts	Gulf stream eddies	

Glacial periods and orbital eccentricity

M. Milankovich (1930)





$7-8^0 C$

$J_E \sim 1/r^2$

$\frac{\delta J_E}{J_E} \sim 10^{-3}$

$\delta T_F \approx 0.07^0 C$

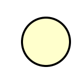
$\frac{\delta T_F}{T_F} \approx \frac{1}{4}\frac{\delta J_E}{J_E}$

<u>Problems:</u>

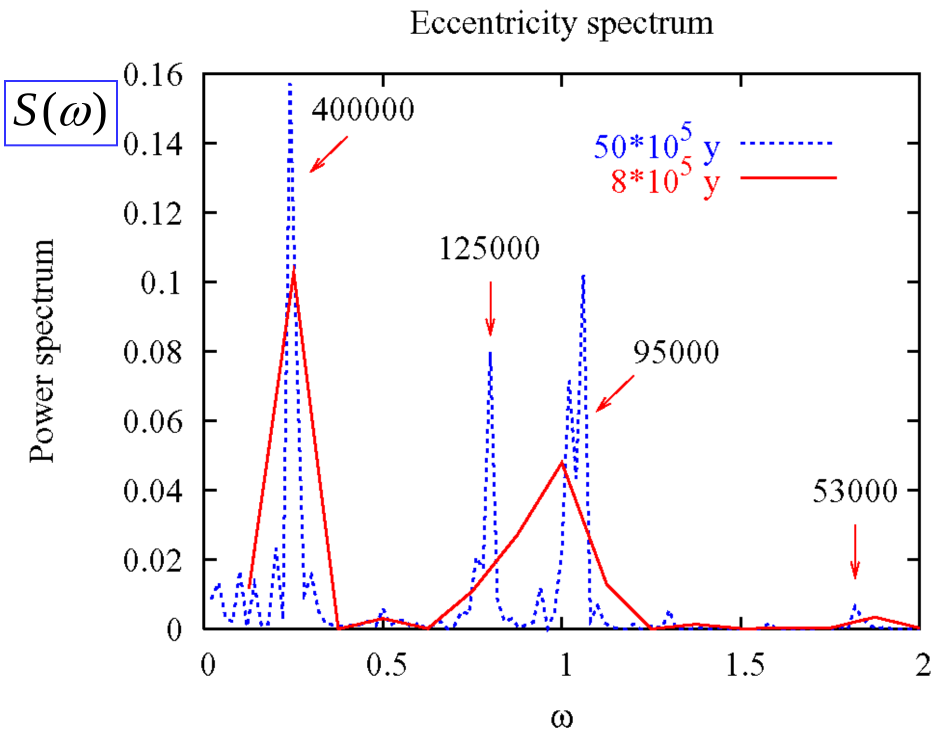
(1) Two orders of magnitude missing
(2) 400 ky period

$$J_E \approx a T_F^{\ 4}$$

$$J_E + \delta J_E \approx a(T_F + \delta T_F)^4$$



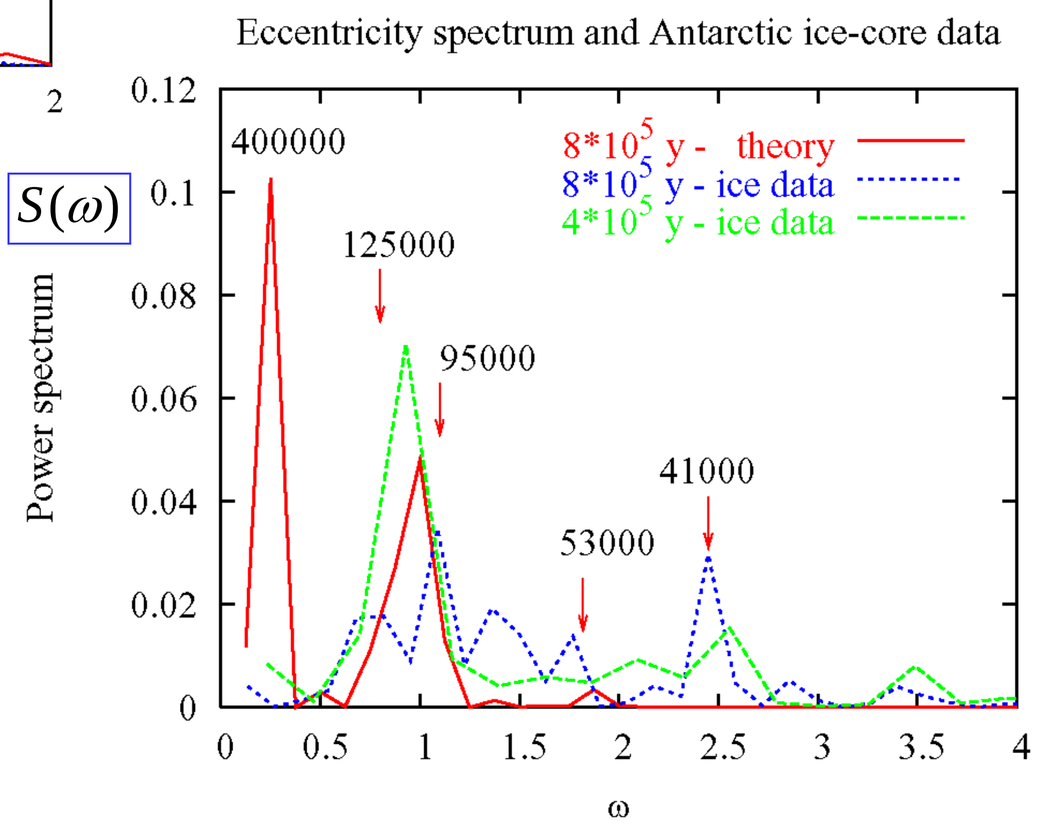
Spectrum of orbital eccentricity



$$f(t) \to F(\omega)$$

Power spectrum:

$$S(\omega) \sim |F(\omega)|^2$$



Problems:

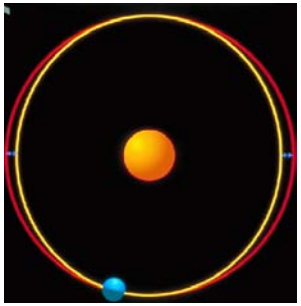
400 ky period missing

100 ky not quite well placed

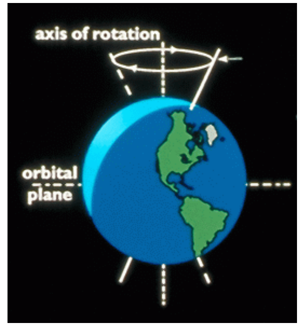
extra periods

Periods of Earth: (Milankovich 1930)

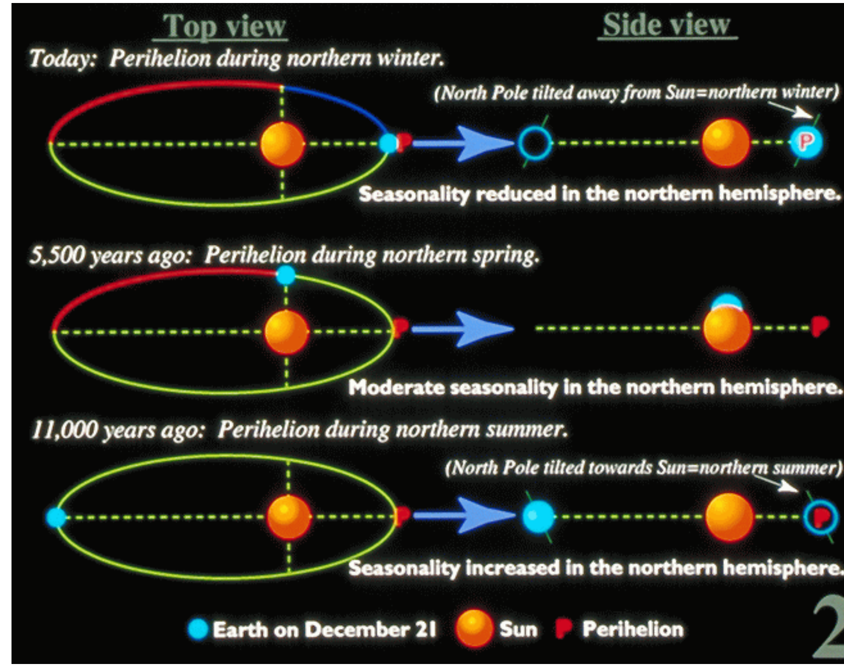
G. Bacsák



Excentricity (100 ky)
small effect – 0.1%

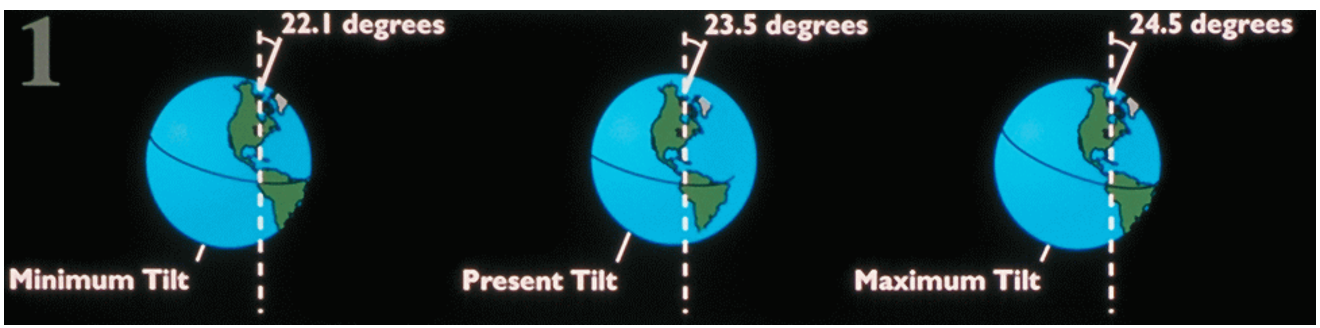


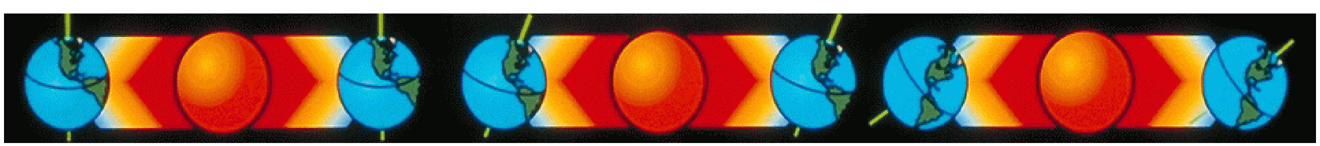
Precession of
axis of rotation:
(19, 23 ky)



Affects intensity of seasons.

Angle of inclination (41 ky)





Changes distribution of insolation.

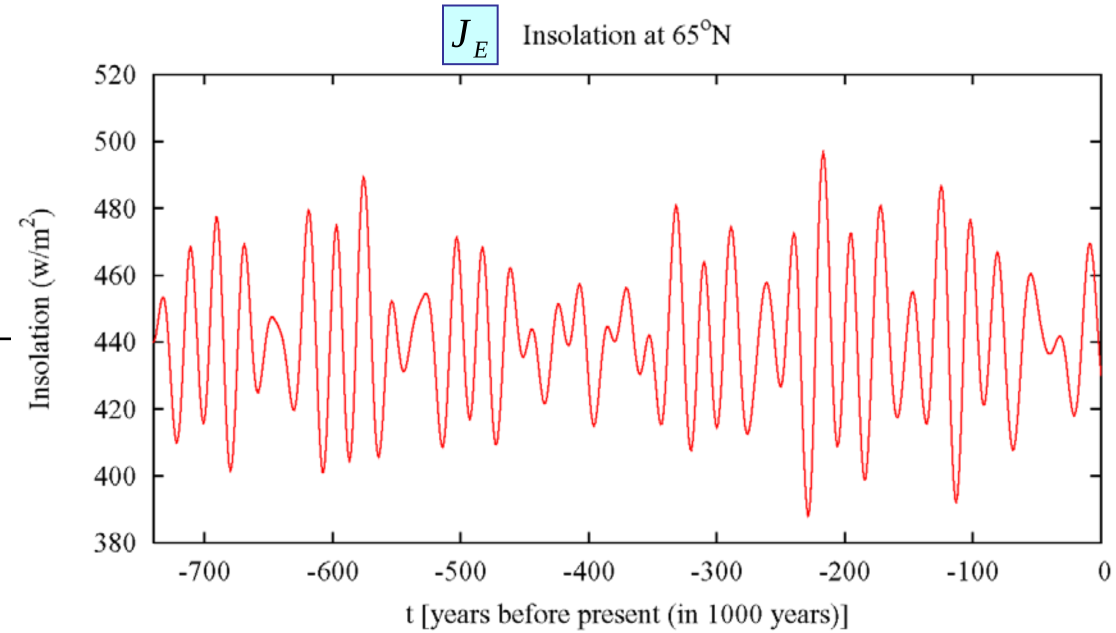
Insolation at North Pole:

max	min
90°	0°

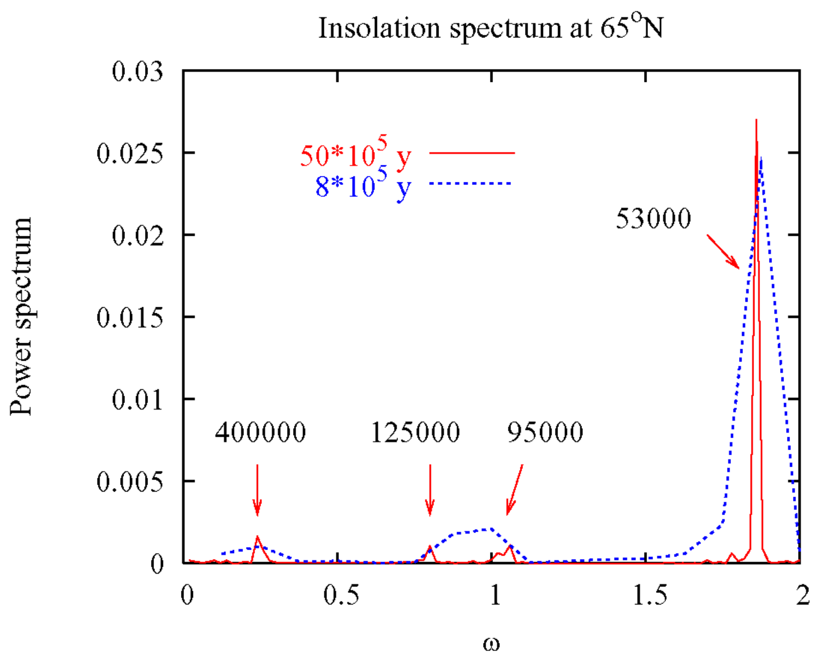
http://www.ncdc.noaa.gov/paleo/

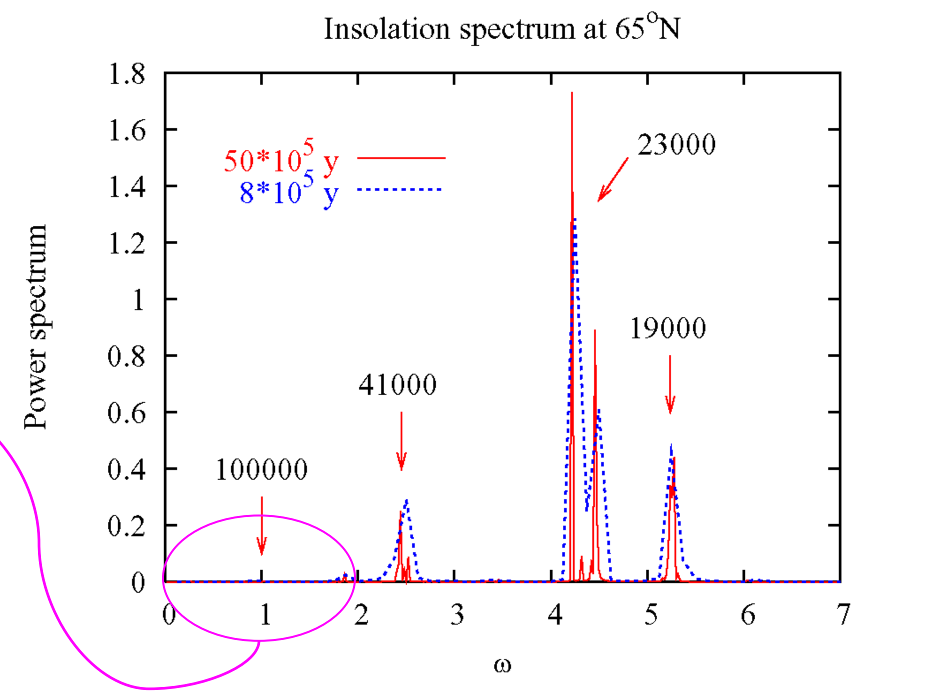
Insolation intensity at the edges of the icefields

$\delta T_F \approx 7^0 C$ ← $\frac{\delta J_E}{J_E} \sim 0.1$ ←



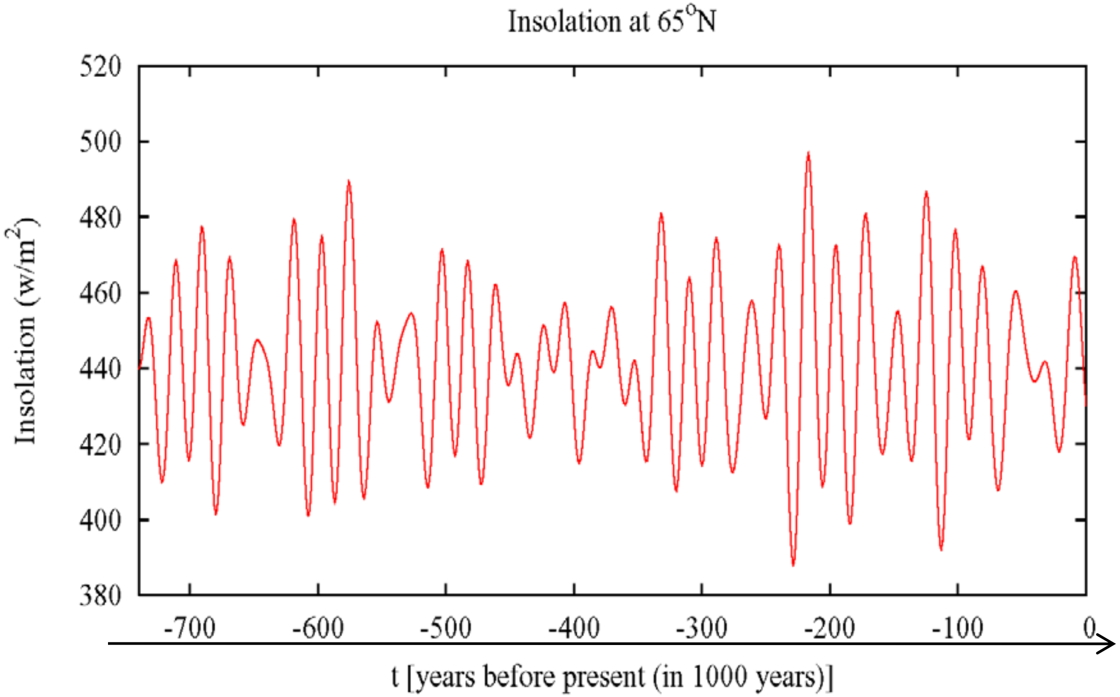
How do we get a 100 ky period from J_E?





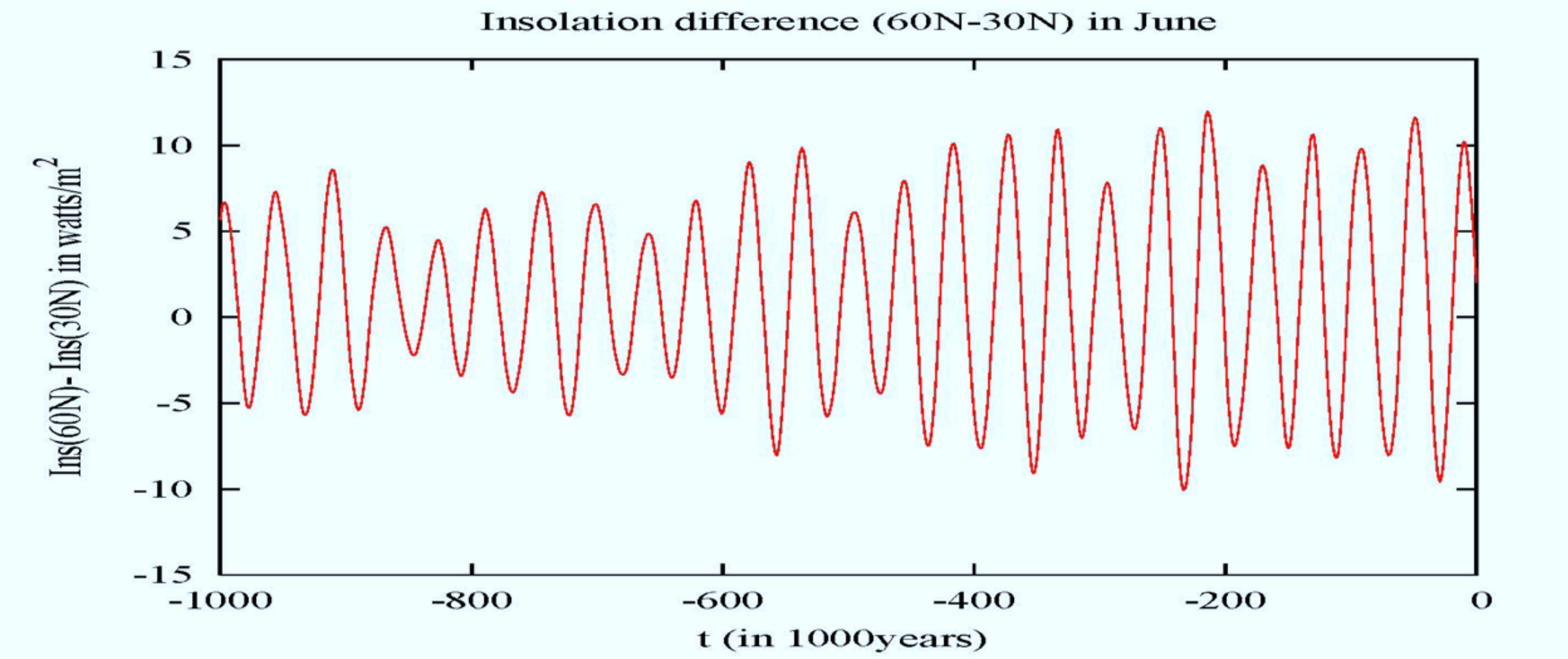
Insolations and insolation differences

$65^0 N$



t

$65^0 N - 30^0 N$



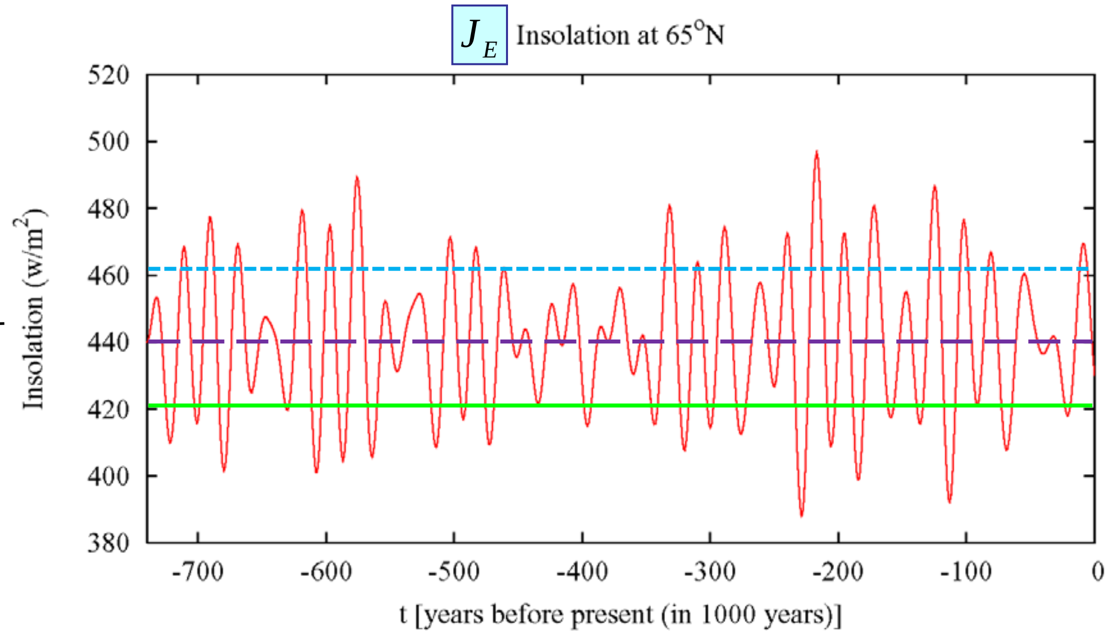
Threshold models

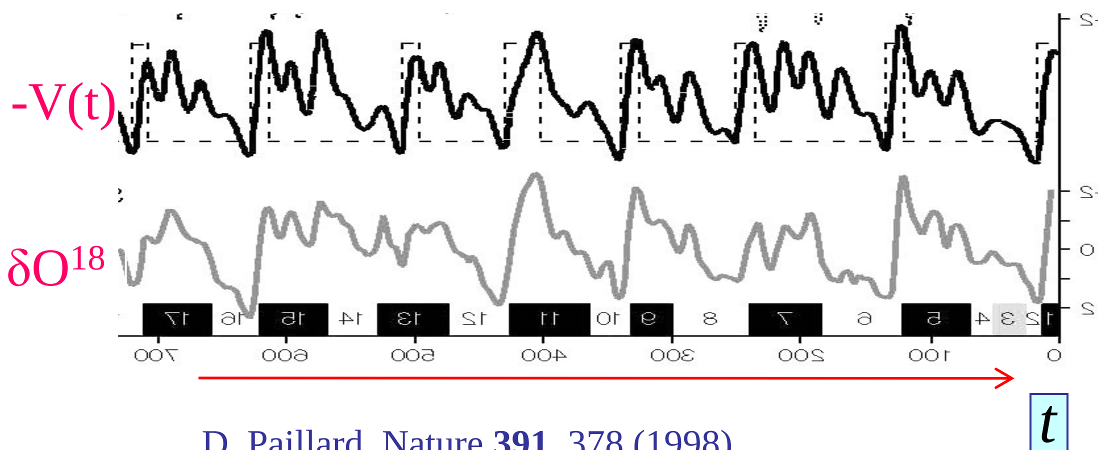
J. Imbrie and J.Z. Imbrie, Science. **207**, 943 (1980)

Complexity of models:
Number of parameters
in data + model

$$I(t) = \frac{J_E - \langle J_E \rangle}{J_E^{\max} - \langle J_E \rangle}$$

$$\frac{dV}{dt} = \begin{cases} \frac{1}{\tau_1}\left[V - \alpha\, I(t)\right] \\ \frac{1}{\tau_2}\left[V - \alpha\, I(t)\right] \end{cases}$$





D. Paillard, Nature **391**, 378 (1998)

A threshold model in more detail

W. H. Berger, Int. J. Earth Sci. **88**, 305 (1999)

V - ice volume

$$I(t) = \frac{J_E - \langle J_E \rangle}{J_E^{\max} - \langle J_E \rangle}$$

$$\frac{dV(t)}{dt} = r - [I(t)]^a \cdot [V(t)]^b$$

Ice fields grow:

$$r = \frac{1}{\tau}$$

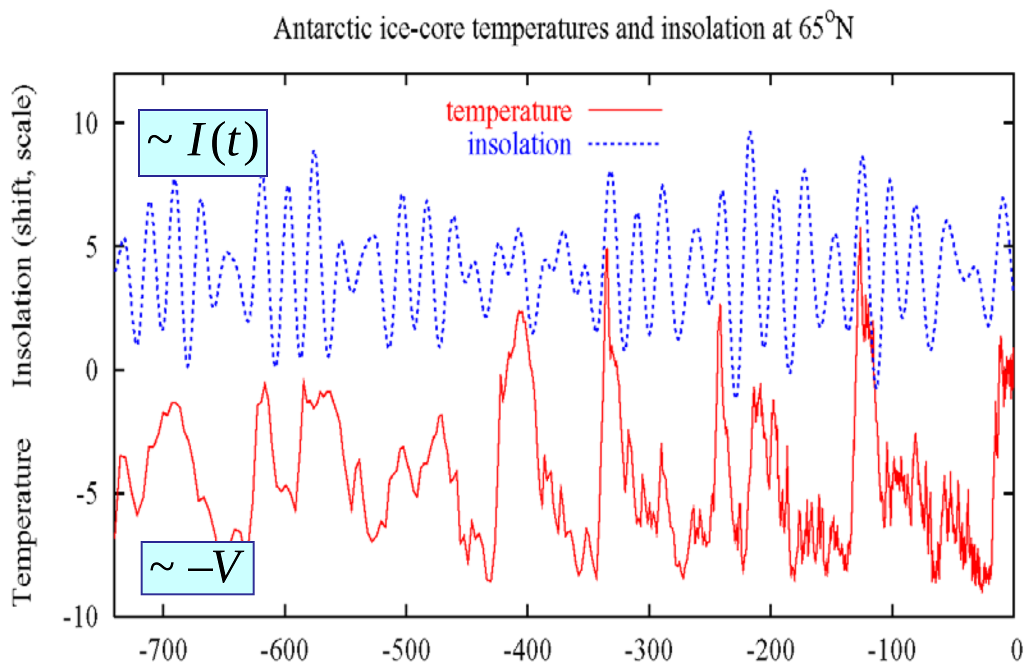
τ ~30 ky

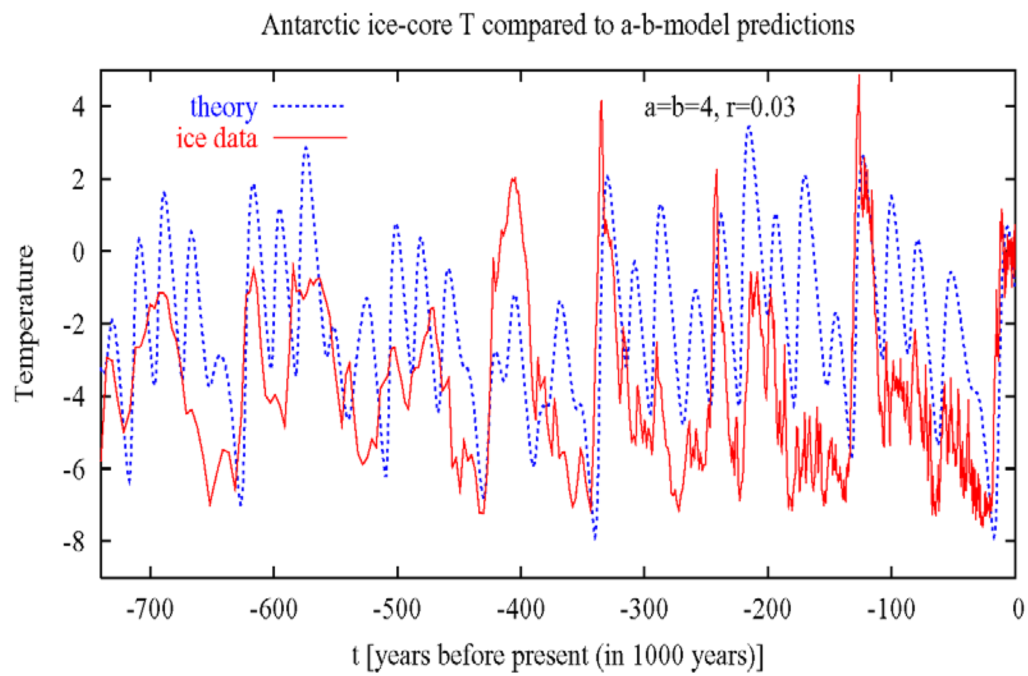
Ice fields unstable if

(1) It is too large (gravitation)

(2) Insolation is large and growing

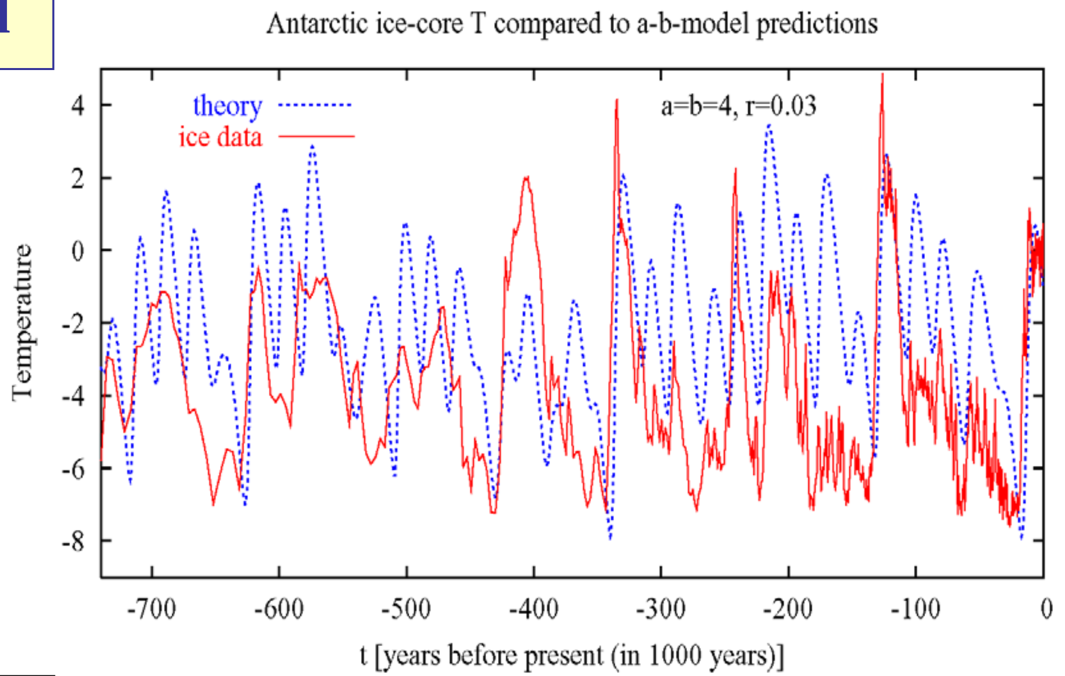
Fitting: $a \approx 4 \quad b \approx 4$





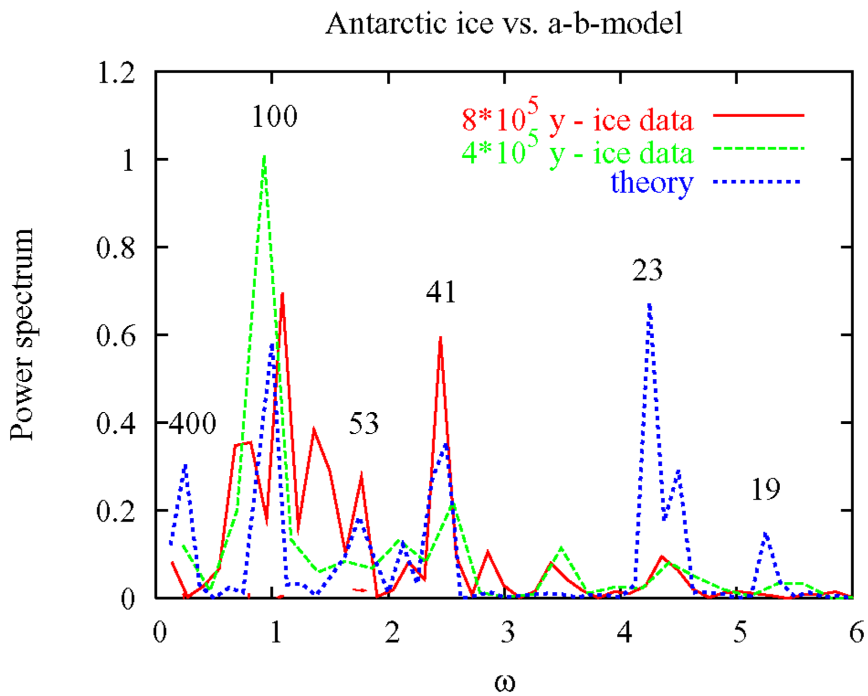
Problems with the treshold model

W. H. Berger, Int. J. Earth Sci. **88**, 305 (1999)



Power spectrum:

$$S(\omega) \sim |F(\omega)|^2$$



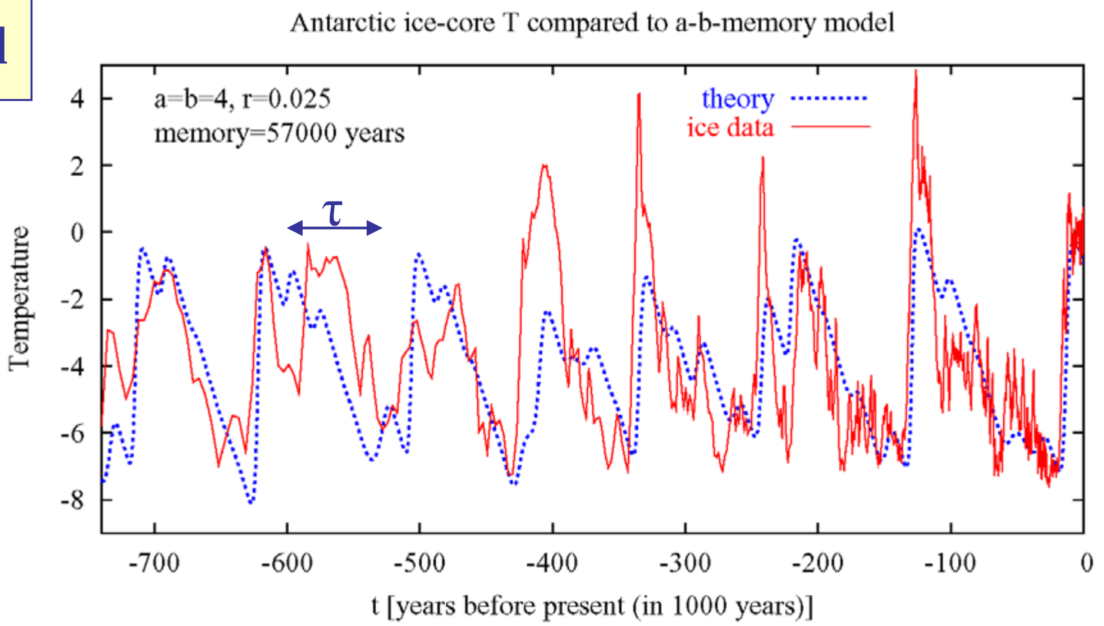
Problems:

400 ky period missing

extra frequencies

Improving the threshold model

W. H. Berger, Int. J. Earth Sci. **88**, 305 (1999)



Memory effects

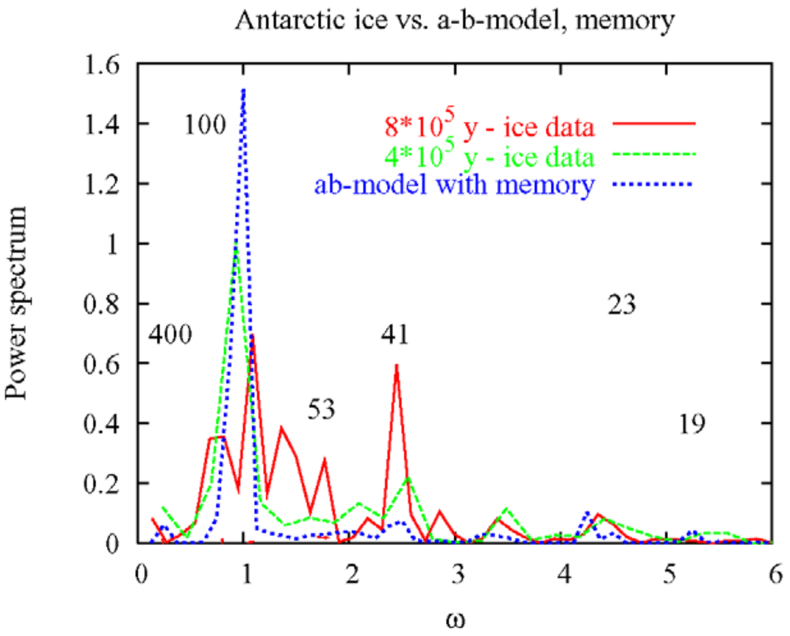
(Effects of the ice flields?)

Average ice volume in the last τ years:

$$\bar{V}(t) = \frac{1}{\tau}\int_{t-\tau}^{t} V(t')dt'$$

$$\frac{dV(t)}{dt} = r - [I(t)]^a \cdot V(t) \cdot [\bar{V}(t)]^b$$

Fit: $\tau \approx 57\ \text{ky}$



Internal drive:
Feedbacks and oscillations

E. Kallen, C. Crafoord, and M. Ghil, J. Atm. Sci. **36**, 2292 (1979)

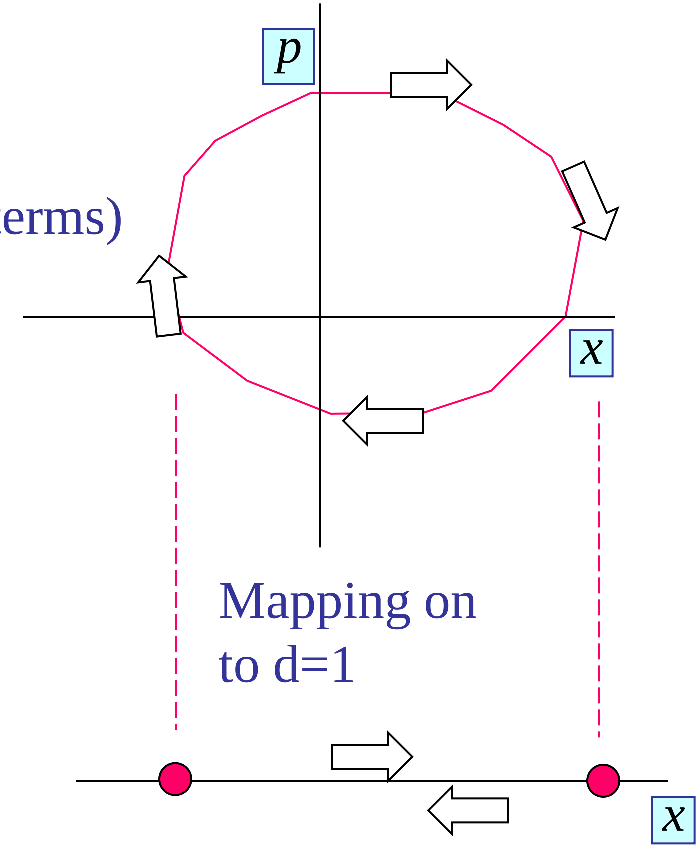
B. Saltzman and A. Sutera, J. Atm. Sci. **41**, 736 (1983)

H. Gildor and E. Tziperman, J. Geophys. Res. **106,** 9117 (2001)

How to get oscillations?

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \Rightarrow \quad \frac{dx}{dt} = p, \quad \frac{dp}{dt} = -\omega^2 x$$

(+ nonlinear terms)



Example:

Temperature (T) → Precipitation (P) → Ice (V) → Albedo (α): → Temperature (T)

Thresholds and/or memory is needed.

Sea-ice switch

H. Gildor and E. Tziperman, J. Geophys. Res. **106,** 9117 (2001)

Box model for T_{land}, T_{sea}, V_{land}, V_{sea}

Warm — sea - ice off

Temperature (T) ⇒ Precipitation (P) — rate: $\approx M_{max}$

⇓

Albedo (α): ⇐ Ice (V) — slow rate: $r_g = M_{max} - S$

⇓

model for growth (M) and ablation (S)

Cold — sea - ice on

Temperature (T) ⇒ Precipitation (P) — rate: $\approx M_{min}$

⇓

Albedo (α): ⇐ Ice (V) — fast rate: $r_d = S - M_{min}$

⇓

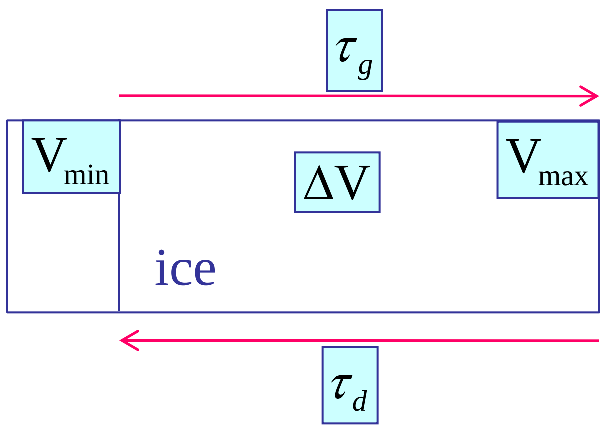
Sea-ice switch: 100 ky period

H. Gildor and E. Tziperman,
J. Geophys. Res. **106,** 9117 (2001)

Rate of growth of ice-shields: $r_g = M_{max} - S$ ← ablation rate

Rate of decay: $r_d = S - M_{min}$ ←

maximal
minimal precipitation rate



Period: $$\tau = \tau_g + \tau_d = \frac{\Delta V}{M_{max} - S} + \frac{\Delta V}{S - M_{min}}$$

hard to determine
but use $\frac{\tau_d}{\tau_g} \approx 0.8$

$$\Delta V = 2.4 \cdot 10^{16} m^3$$

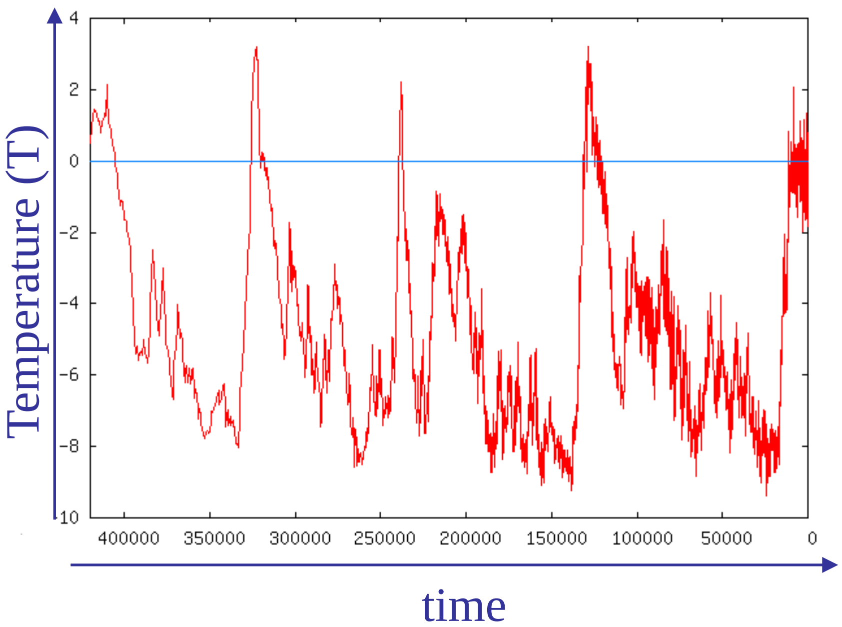
$$M_{max} = 9 \cdot 10^4 m^3 / s$$

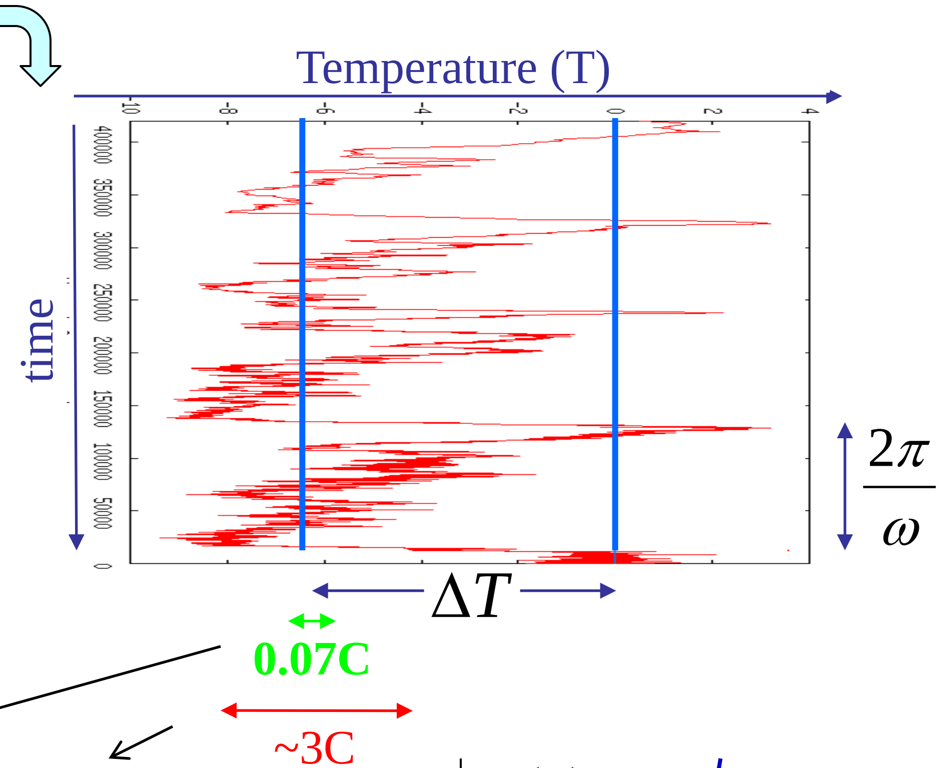
$$M_{min} = 3 \cdot 10^4 m^3 / s$$

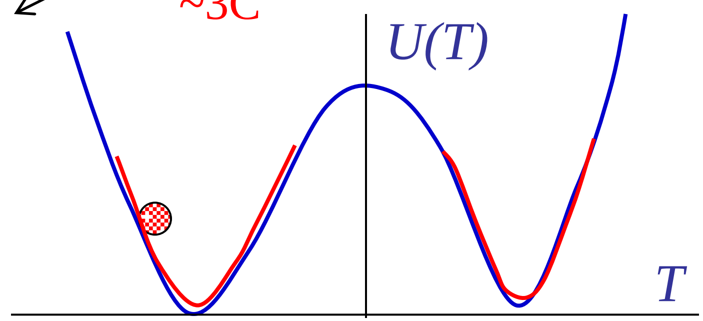
$$\tau = 100 \pm 20 \text{ ky}$$

Stochastic resonance and the 100 ky period

R. Benzi et al., Tellus **34**, 16 (1982), C. Nicolis, ibid. **34**, 1 (1982)







eccentricity drive

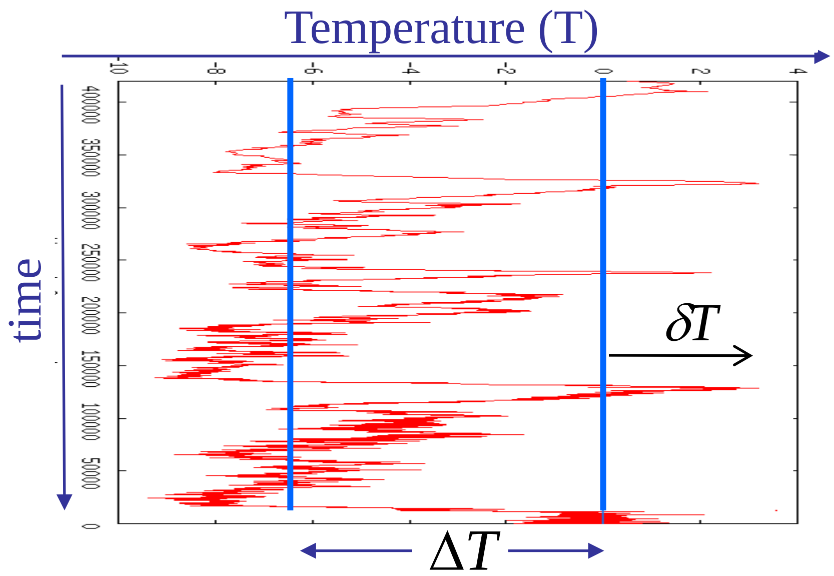
internal noise

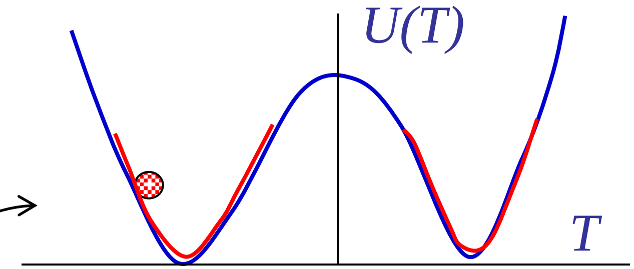
$$\tau_0 \dot{T} = -\frac{\partial U}{\partial T} + a \sin \omega t + \eta$$

timescale

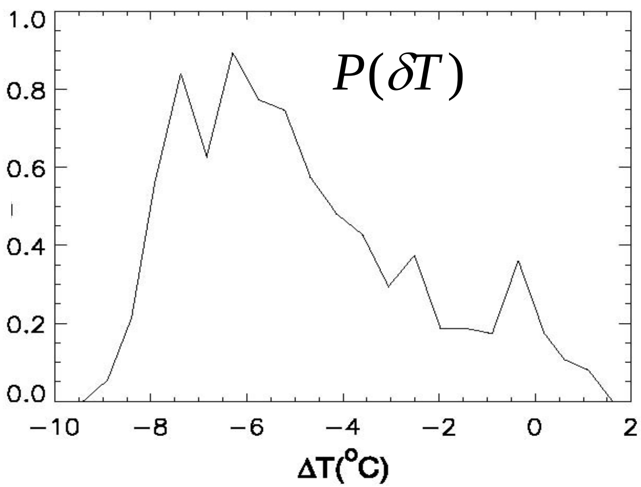
deterministic dynamics

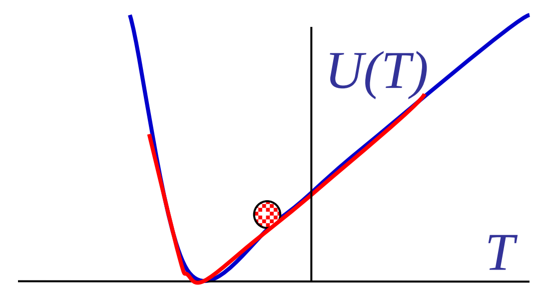
Are there two stationary states?





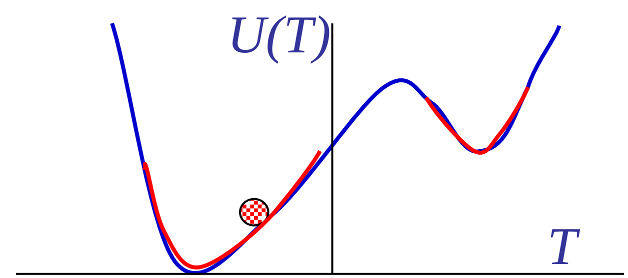
Does it follow? No.





Prob. distribution of T

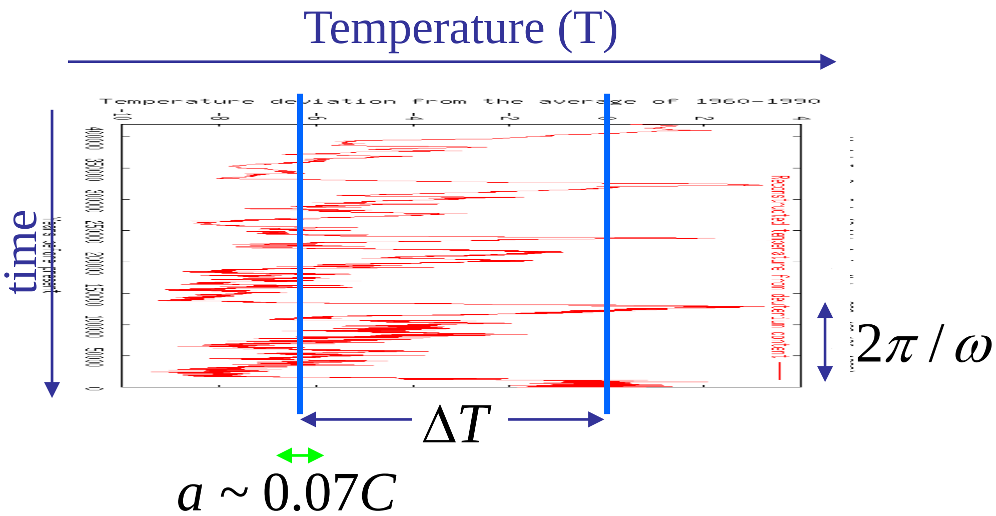
T

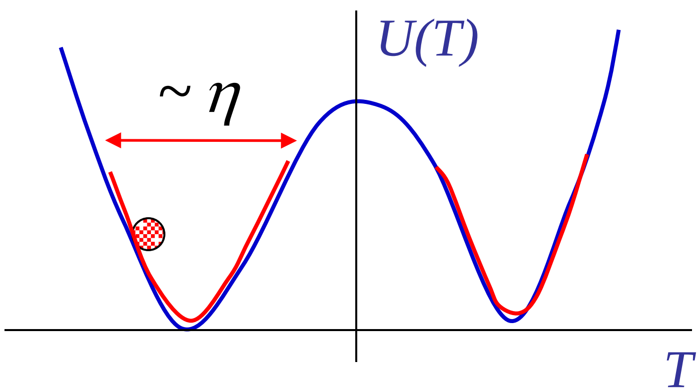


J.D. Pelletier,
J. Geophys. Res. **108**, 4645 (2003)

Stochastic resonance: The mechanism

R. Benzi et al., Tellus **34**, 16 (1982)
C. Nicolis, Tellus **34**, 1 (1982)





$$\tau_0 \dot{T} = -\frac{\partial U}{\partial T} + \eta + a \sin \omega t$$

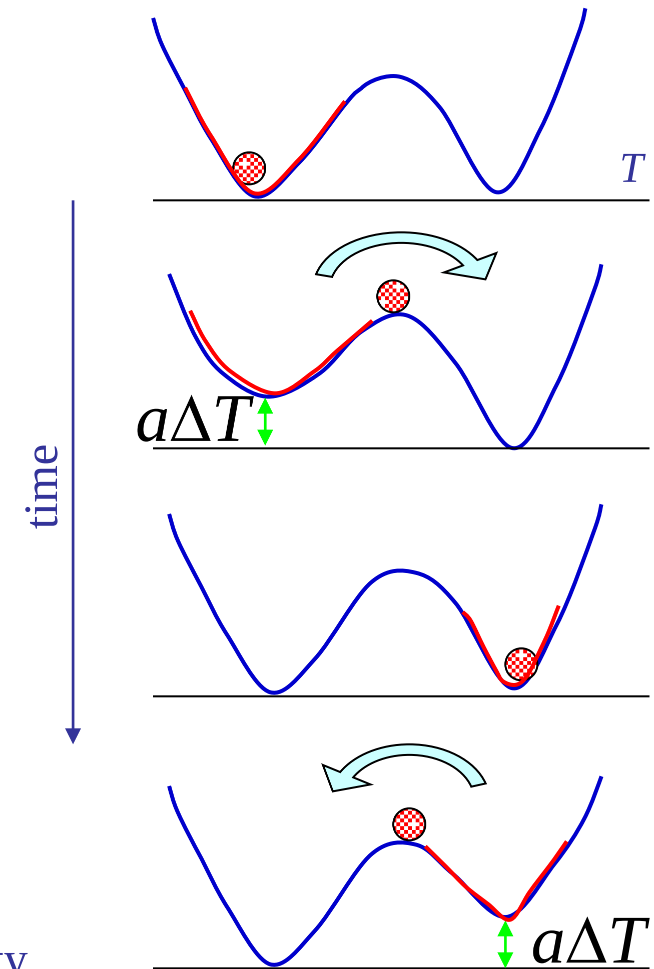
eccentricity

noise

timescale

deterministic dynamics

$$U_e = aT \sin \omega t \qquad \delta U_{e\max} = a\Delta T$$



Derivation of the deterministic part

G. Matteucci
Clim. Dyn. **3**, 179 (1989)

Energy balance:

$$\frac{\partial E}{\partial t} = \mathrm{Q}\left[1 - \alpha(\mathrm{T})\right] - \sigma\,\mathrm{T}^4$$

→ outgoing infrared radiation, parametrized as

$$\sigma T^4 \approx A + B\delta T$$

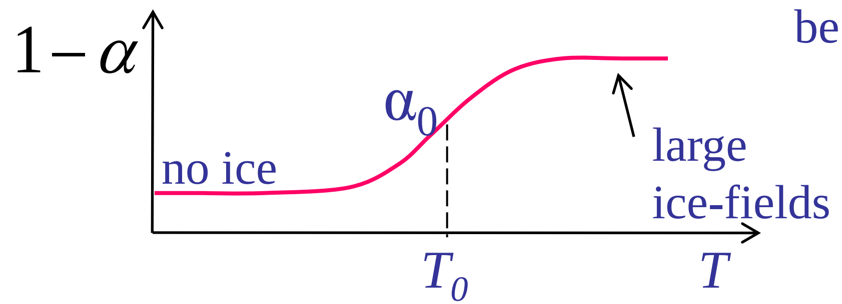
greenhouse may be included here

$$C_E \frac{\partial T}{\partial t}$$

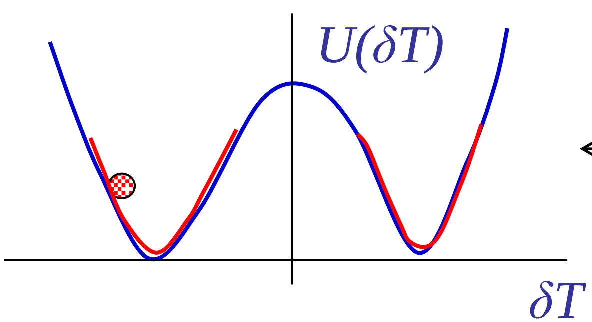
„heat capacity" of Earth

incoming radiation

albedo



$$\alpha(\mathrm{T}) = \alpha_0 - \alpha_1 \tanh\left[(T - T_0)/T_1\right] \approx \alpha_0 - a\delta T + b(\delta T)^3$$



← $\mathrm{r}_0 \approx 0$

$$\frac{\partial \delta T}{\partial t} = \mathrm{r}_0 + \mathrm{r}_1\delta\mathrm{T} - \mathrm{r}_2(\delta T)^3 = -\frac{\partial U}{\partial \delta T}$$

Adding the drive

G. Matteucci, Clim. Dyn. **3**, 179 (1989)

Energy balance:

$$C_E \frac{\partial T}{\partial t} = \mathrm{Q}\left[1 - \alpha(\mathrm{T})\right] - \mathrm{A} - \mathrm{BT}$$

incoming radiation

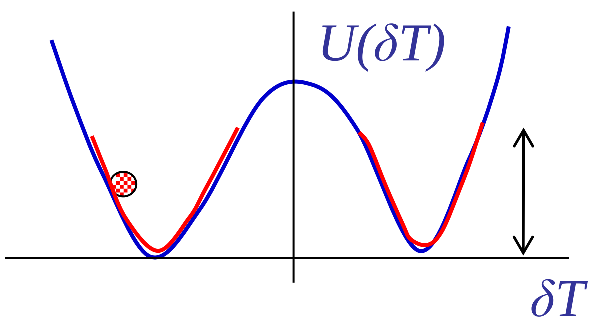
$$\mathrm{Q} = \mathrm{Q}_0 + a\sin(\omega_{ecc} t)$$

$$+\, \mathrm{c}\sin(\omega_{obl} t) \quad \leftarrow \quad \text{nonlinear effect}$$

outgoing infrared radiation may be affected by the seasonality

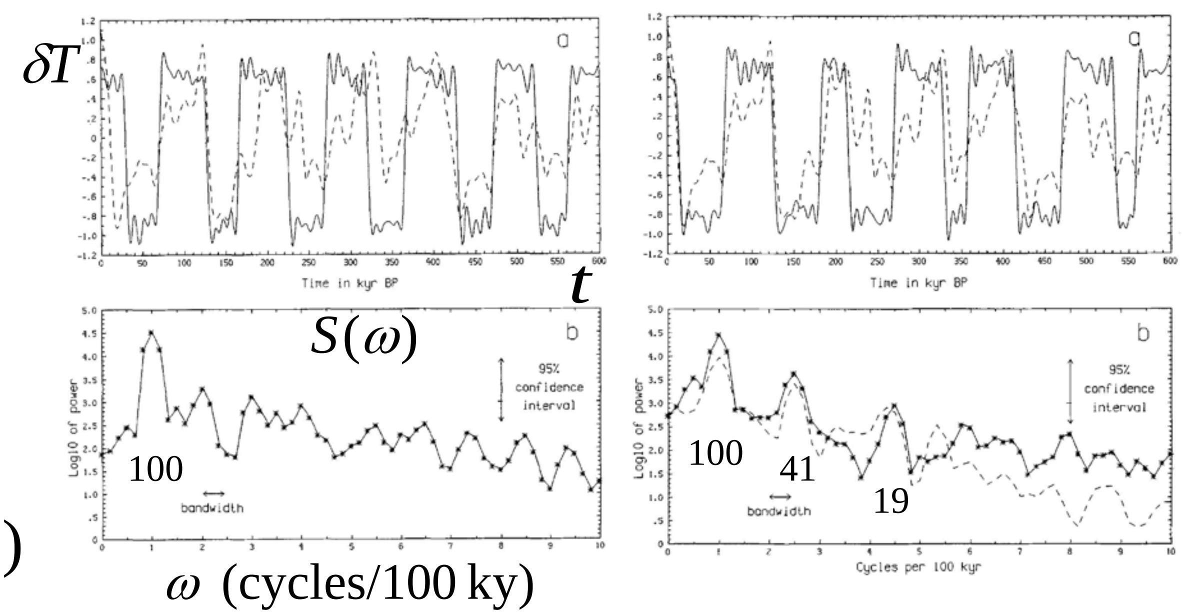
$$\mathrm{B} = \mathrm{B}_0 + b\sin(\omega_{prec} t)$$

Noise:



Gaussian white noise:

$$\langle \eta(t)\eta(t')\rangle = 2D\delta(t-t')$$



Adding memory

J. D. Pelletier, J. Geophys. Res. **108**, 4645 (2003)

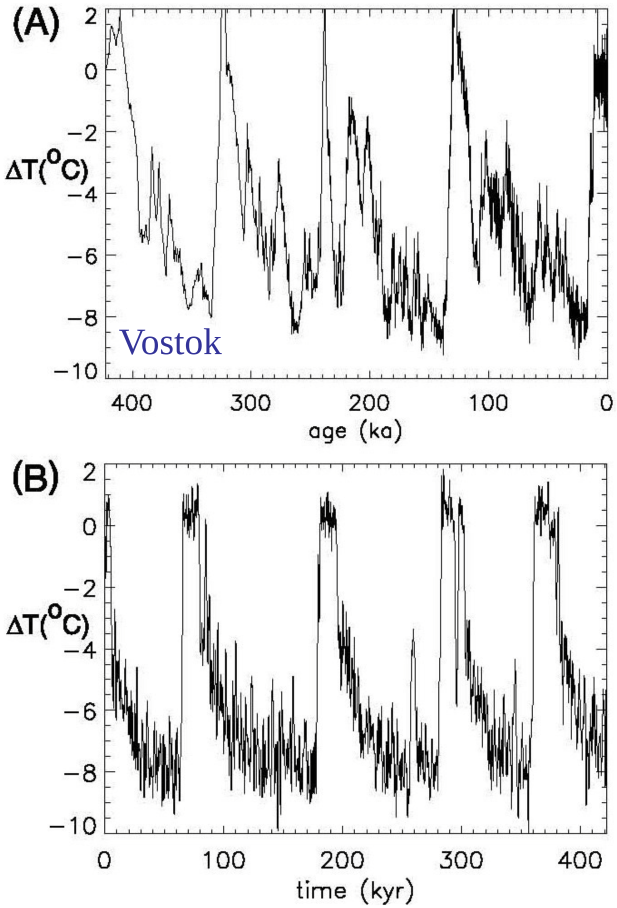
$$\frac{\Delta T_n}{\Delta t} = T_n - T_n^3 + \varepsilon T_{n+\tau} + \eta_n$$

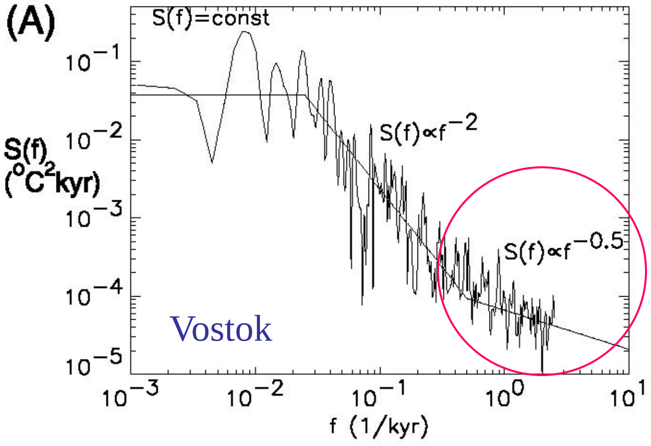
bistability

noise

memory
(lithospheric effects)

solves the problem of directionality





Similar observations:

E. Koscielny-Bunde et al., Phys.Rev.Lett. **81**, 729 (1998)

J. D. Pelletier and D. L. Turcotte, Hydrology, **203**, 198 (1997)

Adding noise to sea-ice switch model

Y. Ashkenazy et al.
J.Geophys.Res. **110**, C02005 (2005)

Dynamics of ice-shields of volume V:

$$\frac{\partial V}{\partial t} = (p_0 - kV)(1 - a_{sea\text{-}ice}) - S + \eta$$

← noise

ablation

Growth slows down when ice volume is large

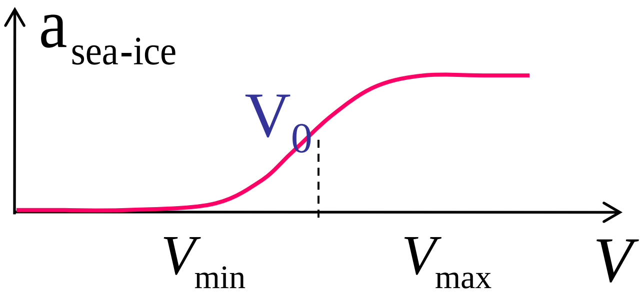
$$a_{sea-ice} = \begin{cases} 0 & \text{off} \\ a & \text{on} \end{cases}$$

$$\frac{\partial E}{\partial t} = Q\left[1 - \alpha(T)\right] - \sigma T^4 + \ldots$$

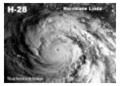
Memory of V_{min} and V_{max} !

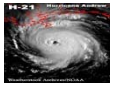
$$\frac{\partial E}{\partial t} = Q\left[1 - \alpha(T)\right](1 - a_{\leftrightarrow}) - \sigma T^4$$

memory term needed for directionality



Do witches exist if there are two large hurricanes in a century?





w: hurricans are caused by witches (idea)

h: more than 2 hurricans occurs in a century (phenomena)

Outset: we do not know

$$P(w) \approx P(\overline{w}) \approx 0.5$$

If w, then the probability of h is big:

$$\mathrm{P}(h \mid w) \approx 0.5$$

If $\overline{w}$, then the probability of h is small:

$$\mathrm{P}(h \mid \overline{w}) \approx 0.1$$

$$P(h, w) = \mathrm{P}(h \mid w)\, P(w) = \mathrm{P}(w \mid h)\, P(h)$$

Probability of h and w

Probability of w if h happens

$$\mathrm{P}(h \mid w)\, P(w) + \mathrm{P}(h \mid \overline{w})\, P(\overline{w})$$

$$\mathrm{P}(w \mid h) = \frac{\mathrm{P}(h \mid w)\, P(w)}{\mathrm{P}(h \mid w)\, P(w) + \mathrm{P}(h \mid \overline{w})\, P(\overline{w})} \approx \frac{0.5}{0.5 + 0.1} \approx 0.83$$