Homework No.9 for Extreme Value Statistics **Deadline May 15th, 4PM.**

(1) 50 pt.

The statistics of extremes are related to the statistics of reaching, for the first time, a threshold value z for a continously changing variable $x(t)$. The solution of the problem below demonstrates this for the case of a random walker.

Let $x(t)$ be the coordinate of a particle which executes a one-dimensional random walk starting from $x(0) = x_0$. The probability $P(x,t)$ that the position of the particle at time t is x satisfies the diffusion equation with some diffusion constant D

$$\partial_t P(x,t) = D\partial_x^2 P(x,t) \quad , \quad P(x,0) = \delta(x - x_0) \quad . \tag{1}$$

When interested in x reaching a treshold value z, one calculates the so called first passage time probability $\hat{\mathbf{P}}(z,t|x_0,0)dt$. It is defined as the probability that the random walker, starting from x_0 at time $t = 0$ passes the $x = z$ value for the first time between t and $t + dt$.

In order to find $\hat{\mathbf{P}}(z,t|x_0,0)$, one can turn to the notes "online additions to Lecture 8" where we calculated the integrated probability, $M(x < z|x_0;t)$, that the random walker, starting from x_0, stayed below $x < z$ up to time t. One should see that $M(x < z|x_0;t)$ is nothing else but 1 minus the probability that the random walker crossed the $x = z$ threshold sometimes in the time interval $[0,t]$. This means that $M(x < z|x_0;t)$ can be expressed through $\hat{\mathbf{P}}(z,t|x_0,0)$ as

$$M(x < z|x_0;t) = 1 - \int_0^t \hat{\mathbf{P}}(z,\tau|x_0,0)d\tau \quad . \tag{2}$$

Use the above equation to calculate the first passage probability $\hat{\mathbf{P}}(z,t|x_0,0)dt$.