Nonequilibrium Phase Transitions

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Outline

(1) Nonequilibrium steady states

Breaking of detailed balance ⇨ problems with usual thermodynamic concepts
Presence of fluxes ⇨ power-law correlations, anisotropy

(2) Phase transitions far from equilibrium

Differences from equilibrium (effects from dynamics)
    Generation of effective long-range interactions, dynamical anisotropies
Searching for nonequilibrium universality classes
    Driven lattice gases, two-temperature models, flocking, …
Where do the power-law correlations come from?
    SOC and absorbing-state transitions, surface fluctuations
Nontrivial distribution functions - using universality

(3) Quantum steady-states with fluxes

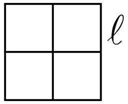
Spin chains with fluxes: T=0 nonequilibrium transitions

(4) Pattern formation

Classification of instabilities;
Real- and complex-coefficient Ginzburg-Landau equations

Equilibrium phase transitions

Statics: Statistical physics → order parameter → scaling → renormalization group

$P \sim e^{-\beta \mathbf{H}}$ M $\chi \sim \xi^{2-\eta}$  $\beta' = f_\ell(\beta)$

Result: Emergence of universality classes d, n, σ

dimension (d), **symmetry of the order parameter** (n), **range of interactions** (σ)

Dynamics near equilibrium: $\tau \sim \xi^{z}$

Master equations - simulations
Langevin equations - field theory

} detailed balance constraint

$$P(t \to \infty) \sim e^{-\beta \mathbf{H}}$$

Result: Dynamical universality classes

d, n, σ

+

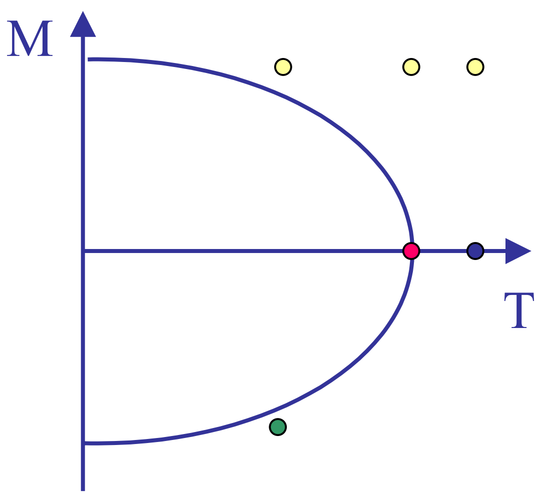
dynamical symmetries

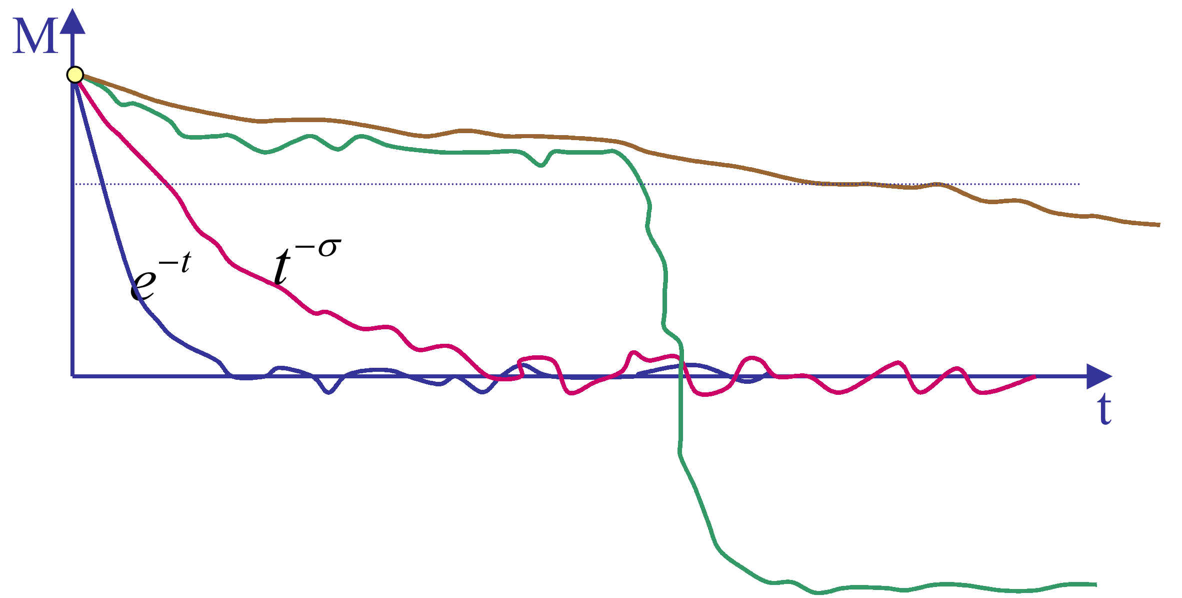
(conservation laws, reversible mode couplings, etc.)

Nonequilibrium steady states

Questions: What are the nonequilibrium steady states?
How to describe them (characteristic features)?
How to construct models which relax to steady states?

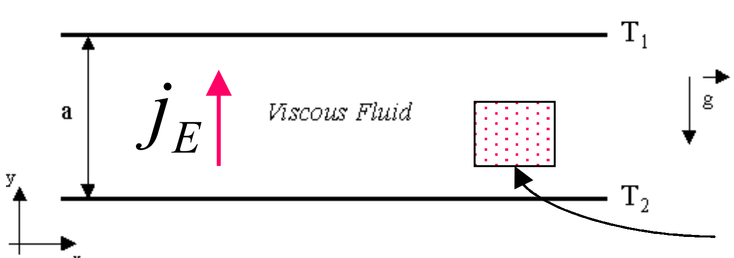
Relaxation to equilibrium

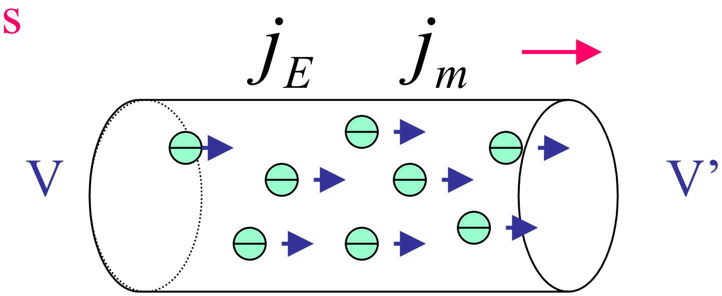




Boundary conditions and drives

fluxes

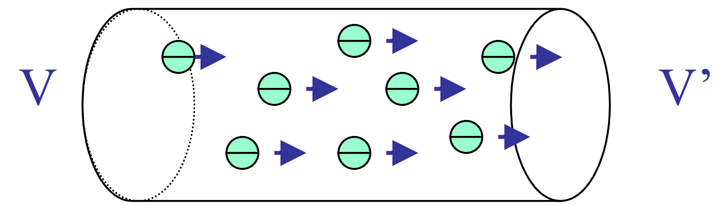




steady state

Nonequilibrium steady states

Simplest case:
(1) fluxes in a homogeneous state
(2) heat baths are in the bulk



Question: How to describe them in terms of simple model systems?

Quasi-microscopic description:

(1) Stochastic process
(2) No memory (markovian process)
(3) Stationary state for $t \to \infty$

$P(s)$ probability of configuration s

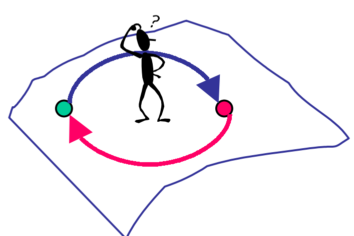
Master equation

$$\partial_t P(s) = \sum_{s'} w_{s\leftarrow s'} P(s') - \sum_{s'} w_{s'\leftarrow s} P(s)$$

Equilibrium: detailed balance

$$w_{s\to s'} P_{eq}(s) = w_{s\to s'} P_{eq}(s)$$

time-reversal symmetry

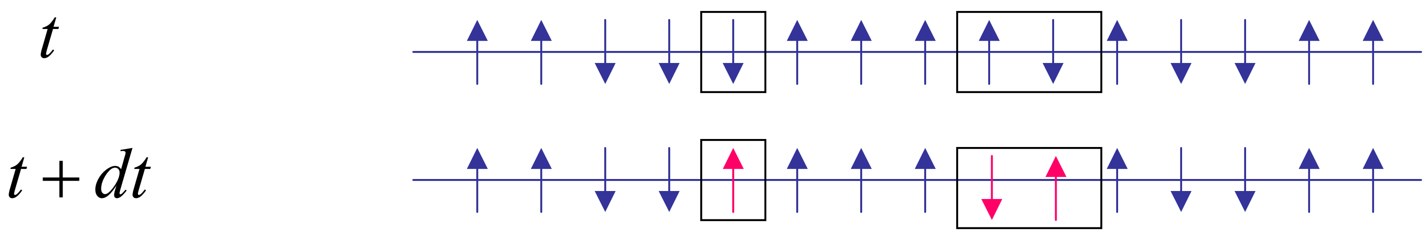


Question: How to break detailed balance?

Breaking detailed balance

Two heat baths

flips: T_1 — $w^{(1)}_{\uparrow\Leftarrow\downarrow}$ — $w^{(2)}_{\downarrow\uparrow\Leftarrow\uparrow\downarrow}$ — Exchanges: T_2



Detailed balance is satisfied for each:

$$w^{(\mathrm{i})}_{s\leftarrow s'} e^{-\beta^{(\mathrm{i})} E_{s'}} = w^{(\mathrm{i})}_{s'\leftarrow s} e^{-\beta^{(\mathrm{i})} E_{s}}$$

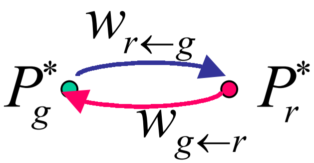
e.g. Metropolis

$$w^{(\mathrm{i})}_{s'\leftarrow s} = \max\{1, e^{-\beta^{(\mathrm{i})}[E_{s'}-E_s]}\}$$

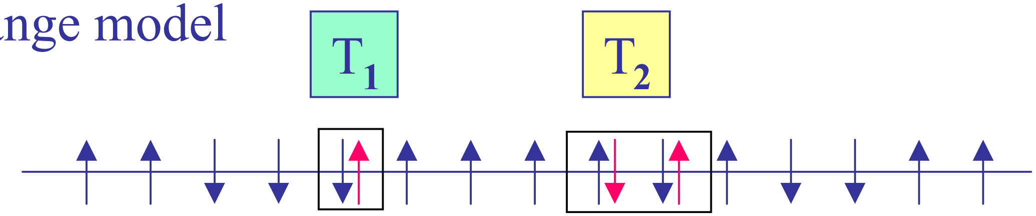
$$\partial_t P(s) = \sum_{\mathrm{i},s'} w^{(\mathrm{i})}_{s\leftarrow s'} P(s') - \sum_{\mathrm{i},s} w^{(\mathrm{i})}_{s'\leftarrow s} P(s) \xrightarrow{t\to\infty} P^*(s)$$

Question: Is detailed balance violated with $P^*(s)$?

Checking the violation of detailed balance

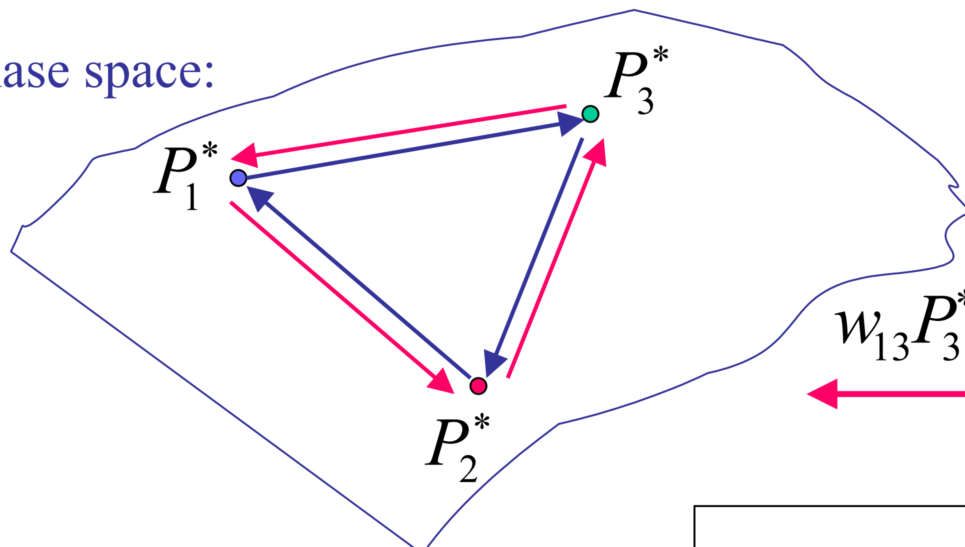


Flip and exchange model



No detailed balance → loops of probability current

Phase space:

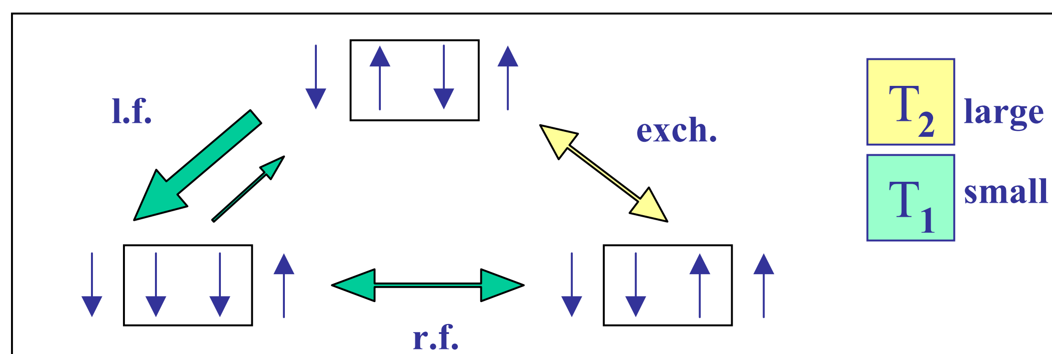


Detailed balance:
all loops carry zero current

$$w_{13}P_3^*w_{32}P_2^*w_{21}P_1^* = w_{12}P_2^*w_{23}P_3^*w_{31}P_1^*$$

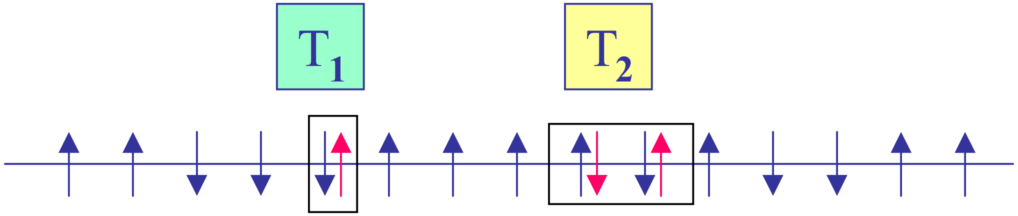
Violation of detailed balance:

$$w_{13}w_{32}w_{21} \neq w_{12}w_{23}w_{31}$$



Effective interactions in the steady state

Flip and exchange



$$P(s,t) \to P^*(s)$$

Steady state:

$$P^*(s) \sim e^{-H_{eff}(s,\beta)}$$

Homework (1)

All interactions allowed by symmetries are generated

$$H_0 = -J \sum_{\langle i,j \rangle} s_i s_j \quad \Rightarrow \quad H_{eff} = -J_2(\beta) \sum_{\{i,j\}} s_i s_j - J_4(\beta) \sum_{\{i,j,k,l\}} s_i s_j s_k s_l - \ldots$$

Ising n.n. interaction

Questions: Can we say something about $J_2(\beta)$, $J_4(\beta)$, ... ?

Is there an effective temperature β_{eff} ?

Methods: (1) expansions (e.g. in β), mean-field theories

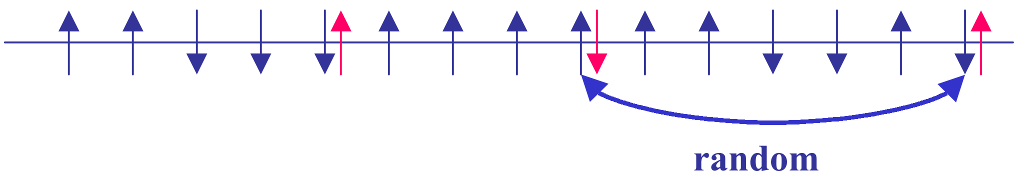
(2) investigate phase transitions + assume universality

⇒ deduce the dominant effective interactions

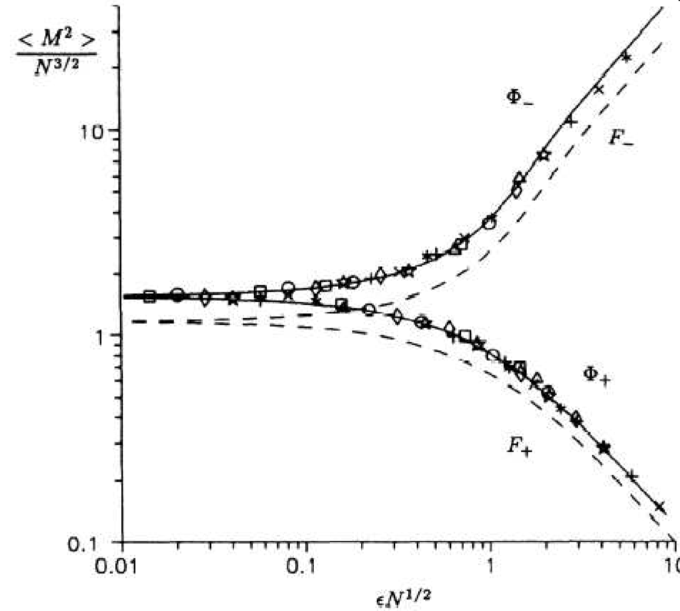
Effective interactions - flip and random exchange

PRA41, 6621 (1990)

control parameter T_1 $T_2 = \infty$

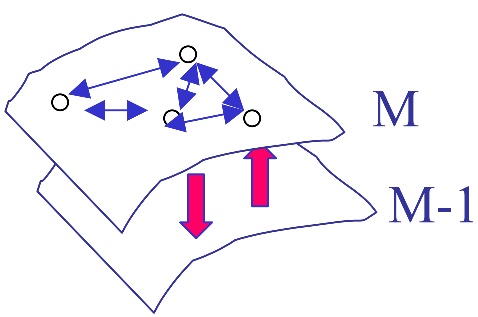


critical magnetization (M) fluctuations



Investigate phase transitions → deduce the dominant effective interactions

$$H_0 = -J \sum_{\langle i,j \rangle} s_i s_j \quad \rightarrow \quad H_{eff} = -\frac{J}{N} \sum_{i,j} s_i s_j$$



Question: Are there detailed balance violation effects?

Restoration of detailed balance in the long-wavelength limit:

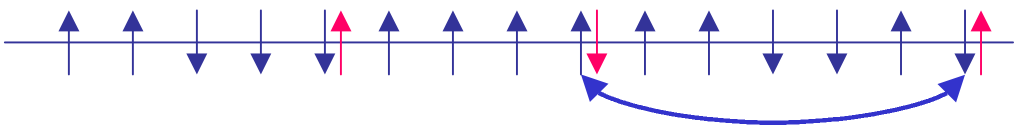
$$w_{M \leftarrow M-1} P^*(M-1) = w_{M-1 \leftarrow M} \; P^*(M)$$

Effective interactions - flip and Levy exchange

PRL67, 3047 (1991)

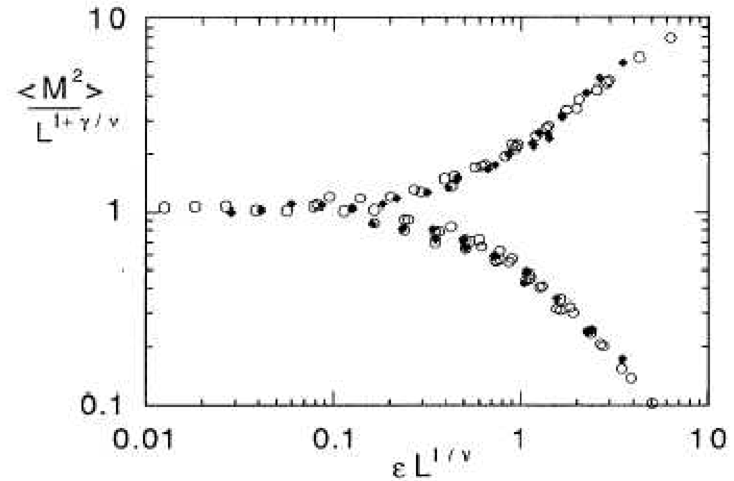
control parameter T_1 $T_2 = \infty$

critical magnetization (M) fluctuations



$$P_{exch}(\ell) \sim 1 / \ell^{1+\zeta}$$

ℓ



Simulations: Critical behavior is compared with the long-range interaction Ising model

$$H_0 = -J \sum_{\langle i,j \rangle} s_i s_j \quad \Rightarrow \quad H_{eff} = -\sum_{i,j} \frac{J}{|i-j|^{1+\sigma}} s_i s_j$$

Result of comparison: $\sigma = \zeta$

Nonequilibrium Levy-flights generate power-law ineractions

Problems with fluctuation-dissipation theorem

Equilibrium, static limit:

$$P_{eq}(M) \sim e^{-\beta \mathbf{H}_0 + \beta H M}$$

independent of H

$$\langle M \rangle = Z^{-1} \sum_{\{\sigma\}} M \, e^{-\beta \mathbf{H}_0 + \beta H M}$$

$$\chi_M = \left. \frac{\partial \langle M \rangle}{\partial H} \right|_{H \to 0} = \beta \langle M^2 \rangle$$

Non-equilibrium steady state:

All interactions allowed by symmetries are generated, expand in $H \to 0$

$$P_{st}(M) \sim e^{-b \mathbf{H}_1 + a H (M + S_3 + \dots)}$$

independent of H

three-spin interactions

$$\chi_M = a \langle M^2 \rangle + a \langle M S_3 \rangle + \dots$$

Ways out:

(1) nonlinear fields

$$Q = M + S_3 + \dots$$

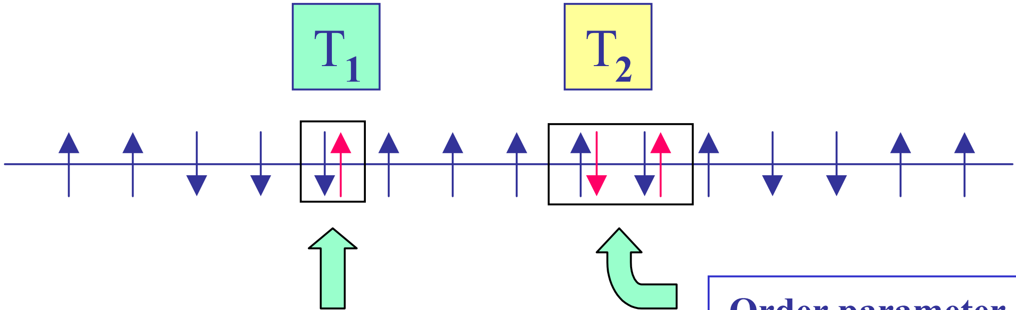
$$\chi_Q = \left. \frac{\partial \langle Q \rangle}{\partial H} \right|_{H \to 0} = a \langle Q^2 \rangle$$

(2) decoupling

$$\langle M S_{2n+1} \rangle \approx \langle M^2 \rangle f_n(C_2)$$

$$\chi_M = a \langle M^2 \rangle \Phi(C_2)$$

Robustness of local relaxational dynamics



Relaxational dynamics: order par. M is not conserved, M relaxes locally (Model A).

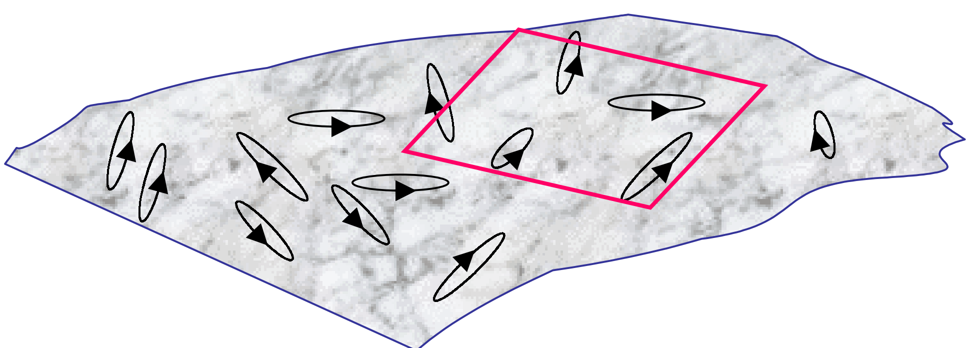
Order parameter conserving (Model B type) perturbation leading to detailed balance violation

Does not change Model A type critical behavior

Model A dynamics is robust against:

(1) Model B type perturbations
(2) Model B type perturbations violating up-down symmetry
(3) Reversible mode coupling to a non-critical conserved field

phase space



Coarse graining eliminates probability fluxes

Spherical limit - competing and anisotropic dynamics

$S_q^{(i)}$ n-component order parameter (i=1,2,…, n $\longrightarrow \infty$)

$$F = \sum_i \int_q (r_0 + q^2) S_q^{(i)} S_{-q}^{(i)} + u \sum_{i,j=1}^{n} \int_{q,q',q''} S_q^{(j)} S_{q'}^{(j)} S_{q''}^{(i)} S_{-q-q'-q''}^{(i)}$$

$\sim \frac{1}{n}$

justified decoupling

$$un \int_k \langle S_k^{(j)} S_{-k}^{(j)} \rangle \sum_{i=1}^{n} \int_q S_q^{(i)} S_{-q}^{(i)}$$

C_k

$$F = \int_q (r_0 + un \int_k C_k + q^2) S_q^{(i)} S_{-q}^{(i)}$$

$$F = \int_q (r + q^2) S_q^{(i)} S_{-q}^{(i)}$$

self-consistency

$$C_k = 1/(r + k^2)$$

Spherical limit - solving the self-consistency equations

$$F = \int_q (r_0 + un\int_k C_k + q^2) S_q^{(i)} S_{-q}^{(i)}$$

$$F = \int_q (r + q^2) S_q^{(i)} S_{-q}^{(i)}$$

$$C_k = 1/(r + k^2)$$

Self-consistency:

$$r = r_0 + un\int_k \frac{1}{r + k^2}$$

Criticality defined through diverging susceptibility:

$$\chi = C_{k=0} = 1/r \sim 1/(r_0 - r_{0c})^\gamma \to \infty$$

$$0 = r_{0c} + un\int_k \frac{1}{k^2}$$

Determining the critical exponent γ :

$$r = r_0 - r_{0c} + un\int_k \left[\frac{1}{r + k^2} - \frac{1}{k^2}\right]$$

$$2 < d < 4$$

$$r \sim (r_0 - r_{0c})^{2/(d-2)}$$

Dynamics in the spherical limit

$$\dot{S}_q^{(i)} = -\Gamma_q \frac{\delta F}{\delta S_{-q}^{(i)}} + \eta_q^{(i)}$$

$$\langle \eta_q^{(i)}(t) \eta_{q'}^{(j)}(t') \rangle = 2\Gamma_q \delta_{i,j} \delta_{q,-q'} \delta(t - t')$$

equilibrium $P\{S\} \sim e^{-F\{S\}}$

$$= -\Gamma_q \left[r_0 + q^2 + u \sum_{j=1}^{n} \int_{q',q''} S_{q'}^{(j)} S_{q''}^{(j)} \right] S_q^{(i)} + \eta_q^{(i)}$$

Model A:

$$\Gamma_q = \Gamma_0$$

Model B:

$$\Gamma_q = Dq^2$$

conserved order parameter

averaging over and initial conditions

$$\langle S_q^{(i)} S_{-q}^{(i)} \rangle_{\eta,0} = C_k(t)$$

$$\dot{S}_q^{(i)} = -\Gamma_q (r_0 + un \int_k C_k + q^2) S_q^{(i)} + \eta_q^{(i)}$$

$$\dot{S}_q^{(i)} = -\Gamma_q(t) S_q^{(i)} + \eta_q^{(i)}$$

solve self consistently

Competing dynamics in the spherical limit

$\dot{S}_q^{(i)} =$ generated by two heat baths at different temperatures

- Relaxational (Model A, spin-flip) dynamics: $\Gamma_q^{\ A} = \Gamma_0$ at $T_A \to r_A$

$$-\Gamma_0\,[r_A + q^2 + u..\text{nonl}..]\,S_q^{(i)} + \eta_{qA}^{(i)}$$

- Diffusive (Model B, spin-exchange) dynamics: $\Gamma_q^{\ B} = Dq^2$ at $T_B \to r_B$

$$-Dq^2\,[r_B + q^2 + u..\text{nonl}..]\,S_q^{(i)} + \eta_{qB}^{(i)}$$

add up

$$\dot{S}_q^{(i)} = -\Gamma_0\left[r_A + a_q q^2 + u_q \sum_{j=1}^{n} \int_{q',q''} S_{q'}^{(j)} S_{q''}^{(j)}\right] S_q^{(i)} + \tilde{\eta}_q^{(i)}$$

$$a_q = 1 + D\,(r_B + q^2)/\Gamma_0 \qquad u_q = u\,(1 + Dq^2/\Gamma_0) \qquad \langle \tilde{\eta}_q^{(i)} \tilde{\eta}_{q'}^{(j)} \rangle = 2\,(\Gamma_0 + Dq^2)\,\delta_{i,j}\,\delta_{q,-q'}\,\delta_{t,t'}$$

q → 0 irrelevant perturbations → **Relaxational dynamics is stable against diffusive**

Competing dynamics - effect of anomalous diffusion I

$\dot{S}_q^{(i)} =$ generated by two heat baths at different temperatures

- Relaxational (Model A, spin-flip) dynamics: $\Gamma_q^{\ A} = \Gamma_0$ at $T_A \to r_A$

$$-\Gamma_0\,[r_A + q^2 + u..\mathrm{nonl}..]\,S_q^{(i)} + \eta_{qA}^{(i)}$$

- Anomalous diffusion (Levy-flight exchanges): $\Gamma_q^{\ B} = Dq^{\sigma} \quad \sigma \le 2$ at $T_B \to \infty$

$$-Dq^{\sigma} S_q^{(i)} + \eta_q^{(i)}$$

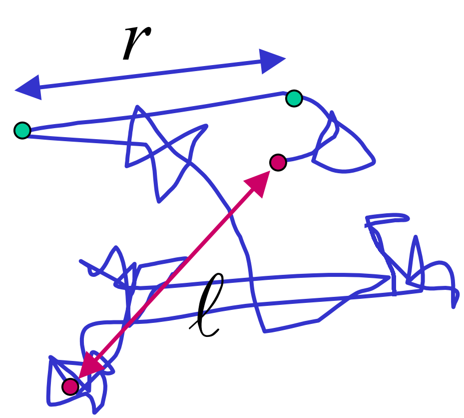
$$\langle \eta_q^{(i)}(t)\eta_{q'}^{(j)}(t')\rangle = 2Dq^{\sigma}\delta_{i,j}\delta_{q,-q'}\delta_{t,t'}$$

add up

Diffusion

$$\ell \sim t^{\frac{1}{2}}$$

$$\omega \sim q^2$$



$$P(r) \sim \frac{1}{r^{d+\sigma}}$$

Levy flights

$$\ell \sim t^{\frac{1}{\sigma}} \longrightarrow \omega \sim q^{\sigma}$$

Competing dynamics - effect of anomalous diffusion II

$$\dot{S}_q^{(i)} = \quad -\Gamma_0\,[r_A + q^2 + u..\text{nonl..}]\,S_q^{(i)} + \eta_{qA}^{(i)}$$

spin-flip dynamics at T_A

$$-\,Dq^\sigma S_q^{(i)} + \eta_q^{(i)}$$

Levy-flight exchanges at $T_B \to \infty$

add up

$$\dot{S}_q^{(i)} = -\Gamma_0\left[r_A + \tfrac{D}{\Gamma_0} q^\sigma + q^2 + u\sum_{j=1}^{n}\int_{q',q''} S_{q'}^{(j)} S_{q''}^{(j)}\right] S_q^{(i)} + \tilde{\eta}_q^{(i)}$$

$q \to 0$

$q^{\sigma \; <2}$ relevant perturbation

$$\langle \tilde{\eta}_q^{(i)} \tilde{\eta}_{q'}^{(j)} \rangle = 2\,(\Gamma_0 + Dq^\sigma)\,\delta_{i,j}\,\delta_{q,-q'}\,\delta_{t,t'}$$

irrelevant

$$\dot{S}_q^{(i)} - \Gamma_0\,[r_A + q^\sigma + u..\text{nonl..}]\,S_q^{(i)} + \eta_{qA}^{(i)}$$

Homework (2)

$$V(x) \sim \frac{1}{|x|^{d+\sigma}}$$

power law is determined by the dimension of the Levy flight

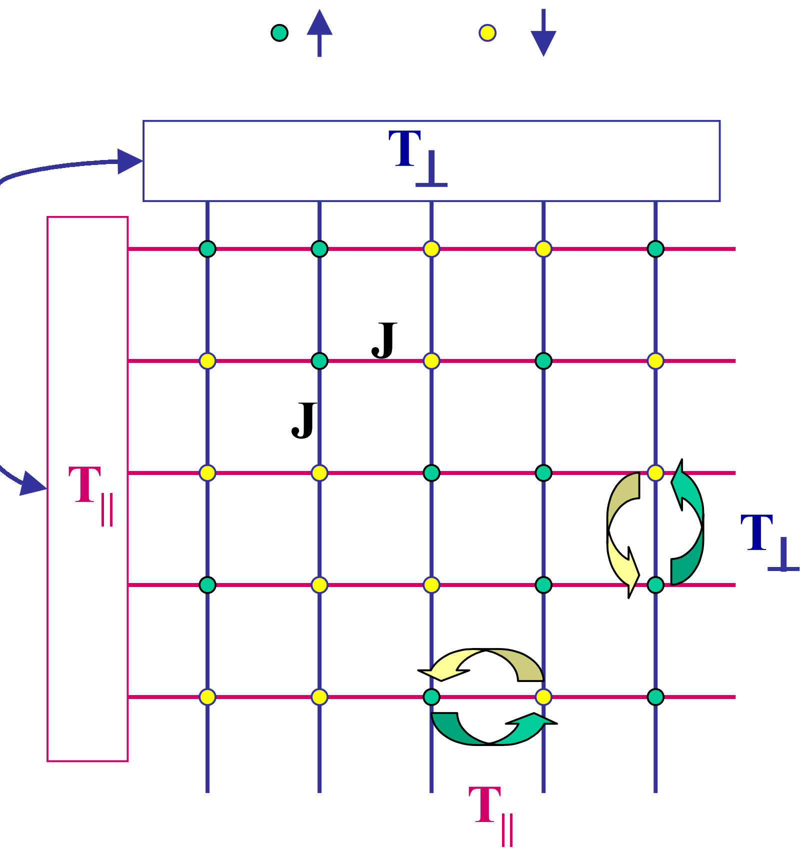
Levy flights generate long-range interactions

Competing dynamics - effects of dynamical anisotropy

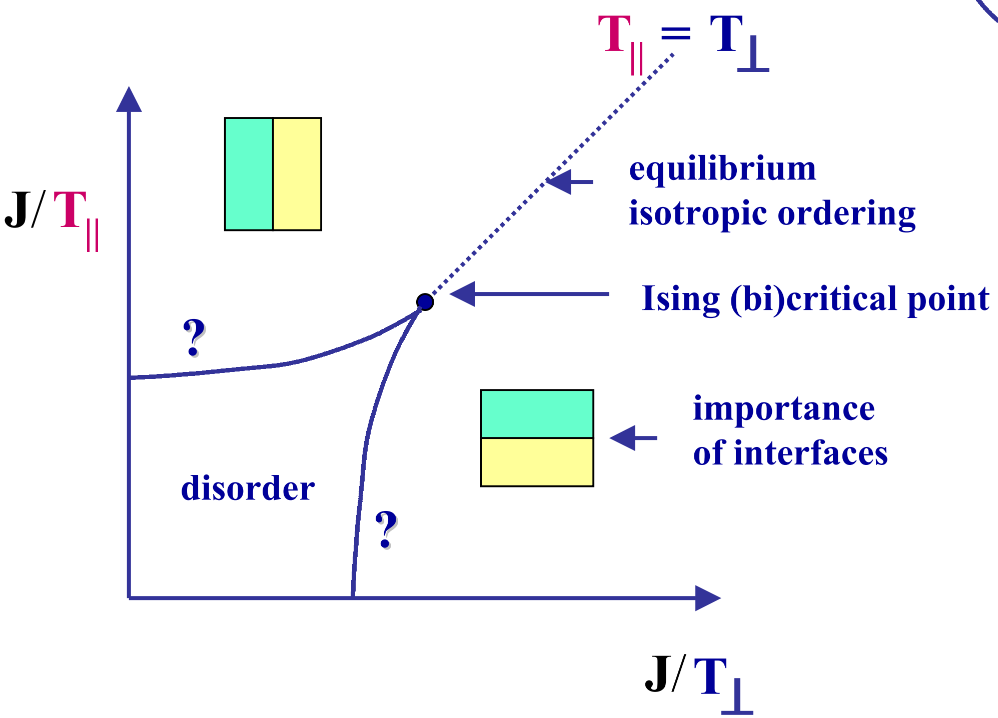
Interaction: Isotropic Ising model (J)

Dynamics: diffusive (spin exch.)

driven by 2 heat baths



Phase diagram



$$\frac{w_{1\to 2}}{w_{2\to 1}} = e^{-\Delta E_{21}/T}$$

same environment - different rates
in the —— and | directions

Competing dynamics - effect of anisotropy II

$$\dot{S}_q^{(i)} = -Dq_{\|}^2\,[r_{0\|} + q^2 + u..\text{nonl}..]\,S_q^{(i)} + \eta_{q_{\|}}^{(i)}$$

exchanges in $\|$ direction

$$-Dq_{\perp}^2\,[r_{0\perp} + q^2 + u..\text{nonl}..]\,S_q^{(i)} + \eta_{q_{\perp}}^{(i)}$$

exchanges in $\perp$ direction

noise correlations are anisotropic

$$\langle \eta_{q_\alpha}^{(i)}(t)\eta_{q'_\alpha}^{(j)}(t')\rangle = 2Dq_\alpha^2\delta_{i,j}\delta_{q,-q'}\delta(t-t')$$

spherical limit:

$$r_{0\|} \to r_{\|} \qquad r_{0\perp} \to r_{\perp}$$

correlations:

$$\langle S_q^{(i)} S_{-q}^{(i)}\rangle = \frac{q_{\|}^2 + q_{\perp}^2}{q_{\|}^2(r_{\|} + q^2) + q_{\perp}^2(r_{\perp} + q^2)}$$

One of the directions becoming soft:

$$r_{\|} \to 0 \qquad r_{\perp} > 0$$

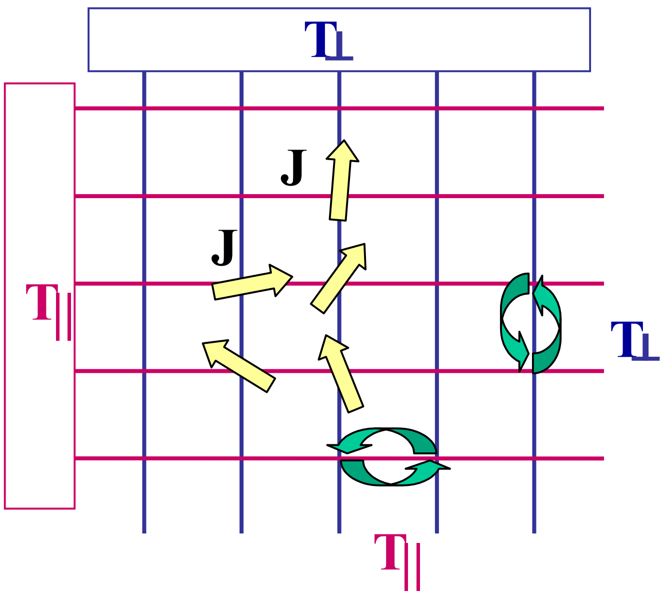
$$\to \frac{1}{q_{\|}^2 + \dfrac{q_{\perp}^2}{q^2} r_{\perp}}$$

Two T diffusive anisotropic dynamics generates quasi-dipole interactions

Competing dynamics - effect of anisotropy III

Long-range order in the d=2 XY model: quasi-dipole interactions are generated by the two-T diffusive anisotropic dynamics

XY spin

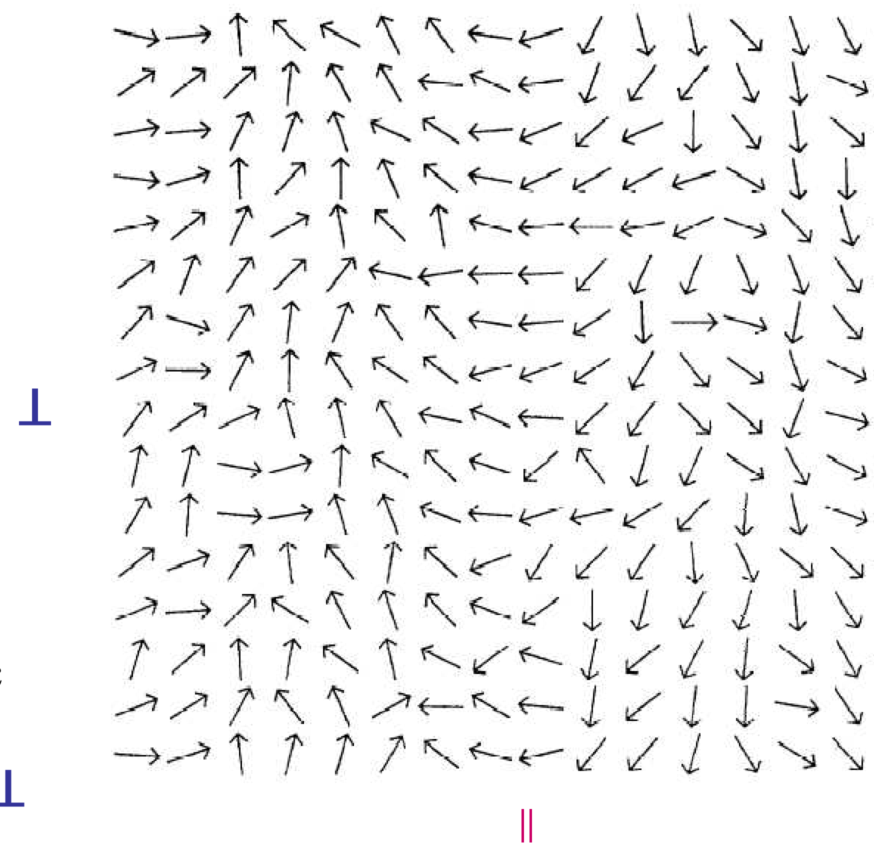


Mermin-Wagner doesn’t work

$T_{||} < T_c$

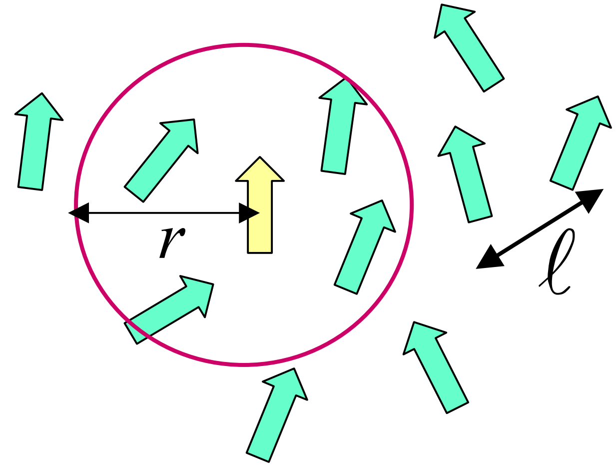
$T_{||} < T_\perp$

simulations

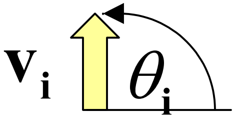


Internal drive: Flocking behavior

T.Vicsek et al. 1995



Driven system: $|\mathbf{v}_i| = 1$



Local interactions: r

Dynamics: follow the neighbors

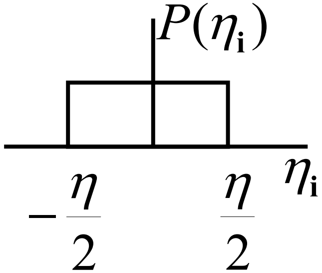
$$\theta_i(t) = \langle \theta(t) \rangle_r + \eta_i$$

← noise (errors)

average in ○

Path:

$$\mathbf{x}_i(t+\tau) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\tau$$



Parameters: $r = 1,\ \tau = 1 \rightarrow$ $1/\ell^2 \sim \bar{\rho}, \quad \eta$

density noise

Flocking behavior - simulations

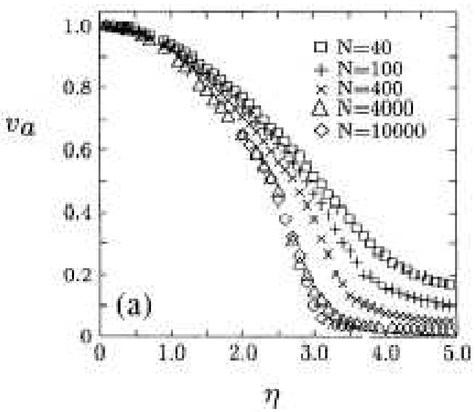
Parameters:

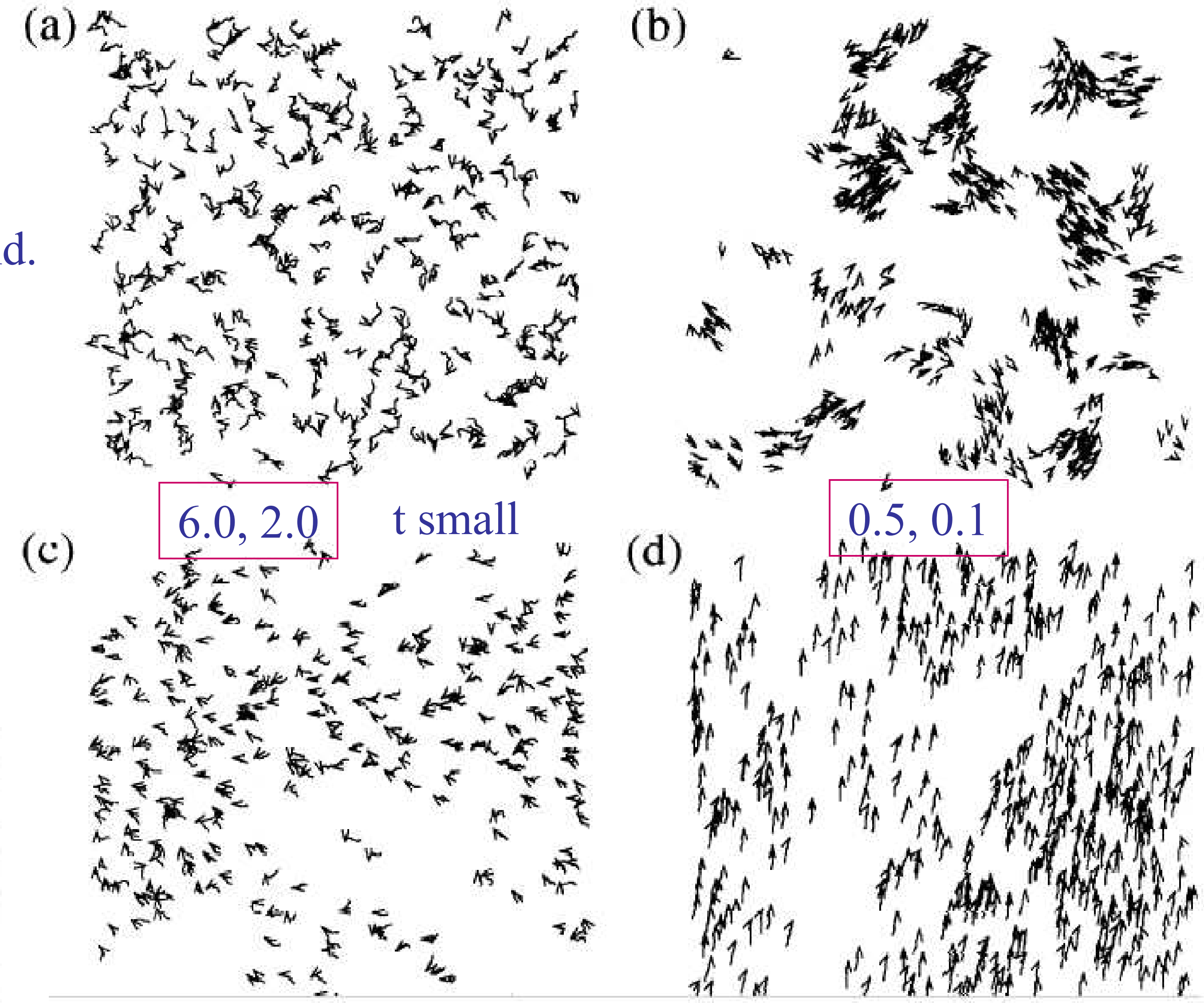
$\bar{\rho}, \quad \eta$

Random initial cond.

Order parameter?

$$v_a = \frac{1}{N}\left|\sum_i \mathbf{v_i}\right|$$





6.0, 2.0 t small 0.5, 0.1

6.0, 2.0 t large 12.0, 0.1

Flocking behavior - field theory

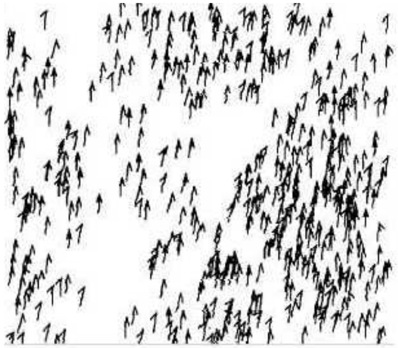
Tu and Toner, 1995

Langevin eq. (hydrodynamics + noise) approach

Fields:

$$\rho(\mathbf{x},t)$$

$$\mathbf{v}(\mathbf{x},t)$$



- Conservation law: birds do not die in flight

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

- Driven system - momentum is generated

$$\partial_t \mathbf{v} + (\mathbf{v}\nabla)\cdot\mathbf{v} = \alpha\mathbf{v} - \beta|\mathbf{v}|^2\mathbf{v} - \nabla P$$

drive saturation

- fixing the 'personal' distance

$$P = \sum_n \sigma_n (\rho - \rho_0)^n$$

$$+ D_1\nabla^2\mathbf{v} + D_2(\mathbf{v}\nabla)^2\cdot\mathbf{v} + D_3\nabla(\nabla\cdot\mathbf{v})$$

dissipative terms

coefficients may depend on ρ and $\mathbf{v}^2$.

$$+\lambda_2(\nabla\cdot\mathbf{v})\mathbf{v} + \lambda_3\nabla\mathbf{v}^2$$

convective-like terms

$$+\eta(\mathbf{x},t)$$

noise (errors)

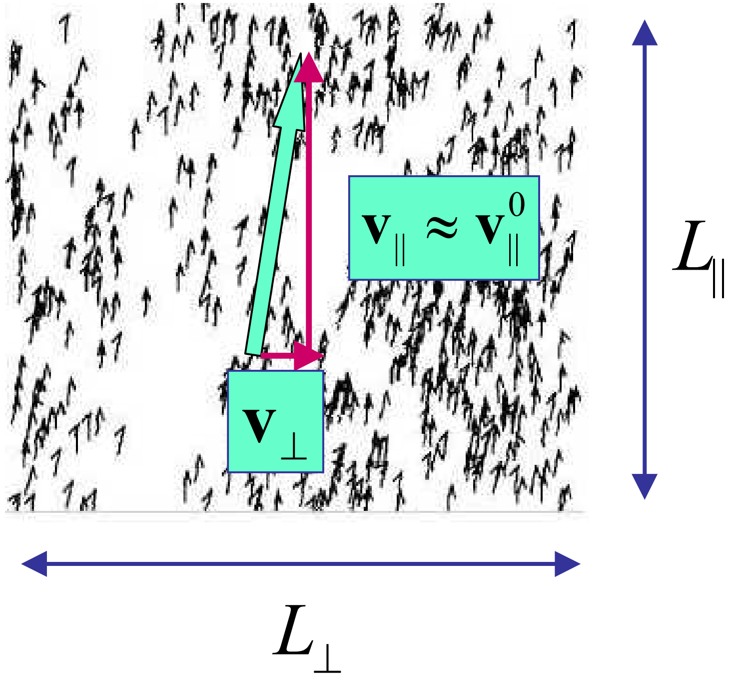
Model A + convection

Flocking behavior - field theory

Tu and Toner, 1995

Broken symmetry phase:

$$\rho = \rho^0 \quad \mathbf{v} = \mathbf{v}_{\parallel}^0$$



Do the fluctuations destroy the ordered state?

$$\rho = \rho_0 + \underline{\delta\rho} \quad \mathbf{v} = \mathbf{v}_{\parallel}^0 + \underline{\mathbf{v}_{\perp}}$$

No, convection is relevant and stabilizes order!

$$\langle |\mathbf{v}_{\perp}|^2 \rangle \approx \text{const} + L_{\perp}^{2\chi} \Phi(L_{\parallel} / L_{\perp}^{\zeta})$$

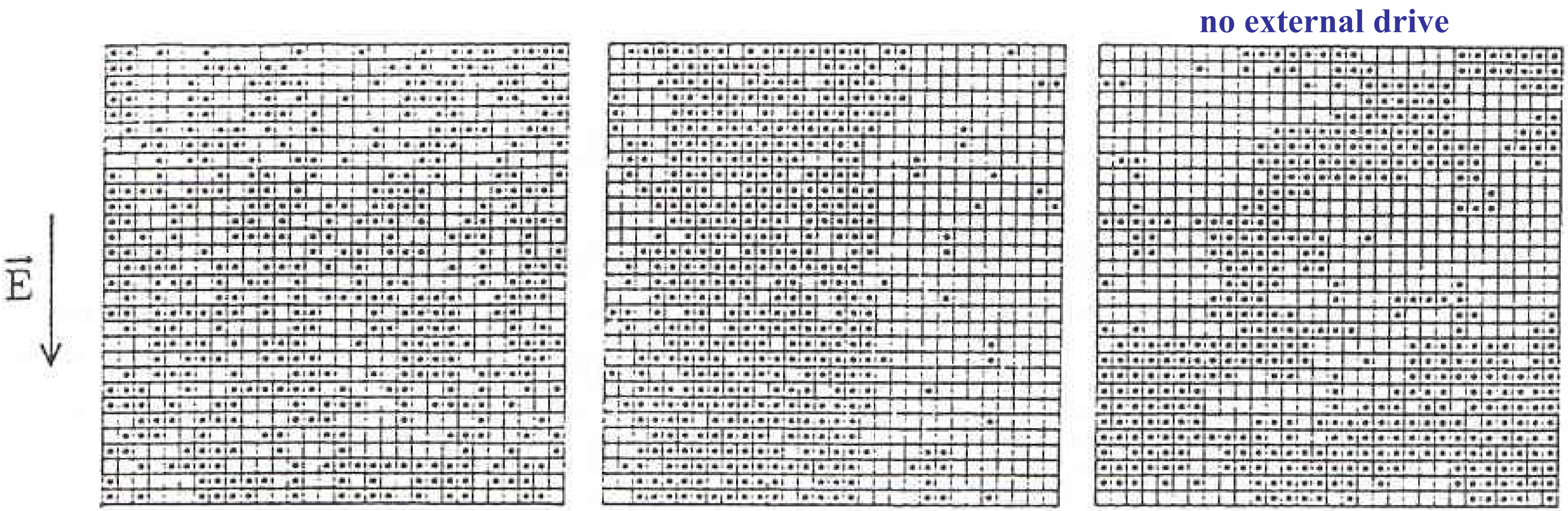
RG, scaling, special symmetry in d=2 →

$$\approx \text{const} + L_{\perp}^{-2/5} \Phi(L_{\parallel} / L_{\perp}^{3/5})$$

Compare: Transverse fluctuations of the magnetization diverge in the d=2 XY model

Driven Ising lattice gases

Katz, Lebowitz and Spohn, 1984



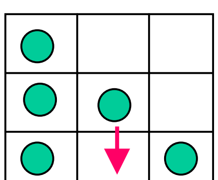
$T > T_c$ $\qquad$ $T < T_c$ $\qquad$ $T < T_c$

Homework(3)

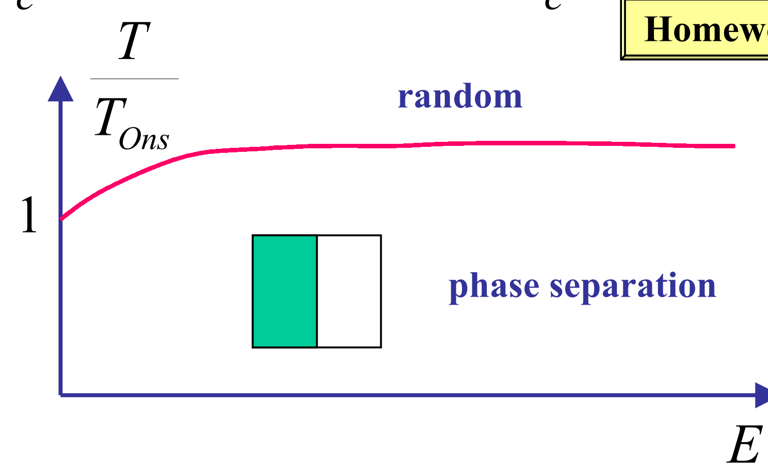
(1) Periodic boundary conditions

(2) Local detailed balance

Moves are calculated using local potential



$$V = Ez$$



Literature

Two-temperature models

J. Marro and R. Dickman

Nonequilibrium Phase Transitions in Lattice models

(Cambridge University Press, Cambridge, 1999)

Driven lattice gases

B. Schmittmann and R.K.P. Zia

in **Phase Transitions and Critical Phenomena, Vol. 17**,

Eds. C. Domb and J.L. Lebowitz (Academic Press, London, 1995)

Latest on field-theoretic results

U. C. Tauber et al.,

Effects of violating detailed balance on critical dynamics

Phys.Rev.Lett. **88,** 045702 (2002)

Short review

Z. Rácz

Nonequilibrium phase transitions

Les Houches Summer School on ‘Slow relaxations and nonequilibrium dynamics in condensed matter’, Eds. J.-L. Barrat et al. (EDP and Springer, New York, 2003)

Finite-size scaling: Critical magnetization fluctuations

Scaling assumption:

$$\langle M^2 \rangle = F(\varepsilon, L) \approx L^{\theta}\Phi(L/\xi) = L^{\theta}\Phi(L\varepsilon^{\nu})$$

$$\varepsilon = (T - T_c)/T_c$$

Asymptotics:

$T > T_c$

$$\langle M^2 \rangle = L^d \chi \approx L^d \varepsilon^{-\gamma} \rightarrow \Phi_+(x) \approx x^{-\gamma/\nu}$$

$$\theta = \frac{\gamma}{\nu} + d$$

$T < T_c$

$$\langle M^2 \rangle = L^{2d} \langle m \rangle^2 \approx L^{2d} |\varepsilon|^{2\beta} \qquad \Phi_-(x) \approx |x|^{2\beta/\nu}$$

$$\theta = -\frac{2\beta}{\nu} + 2d$$

Data analysis:

Plot $\langle M^2 \rangle / L^{d+\gamma/\nu}$ vs. $L\varepsilon^{\nu}$

$$\gamma + 2\beta = \nu d$$

Spherical limit - calculation of the susceptibility exponent

$$\chi = C_{k=0} = 1/r \sim 1/(r_0 - r_{0c})^{\gamma} \to \infty$$

$$r = r_0 + un\int_k \frac{1}{r + k^2}$$

$$0 = r_{0c} + un\int_k \frac{1}{k^2}$$

$$r = r_0 - r_{0c} + un\int_k \left[\frac{1}{r+k^2} - \frac{1}{k^2}\right]$$

$$\int_k [\tfrac{1}{r+k^2} - \tfrac{1}{k^2}] = -rS_d \int_k k^{d-1} dk \, \tfrac{1}{k^2(r+k^2)} = -r^{\frac{d}{2}-1} S_d \int_x dx \, \tfrac{x^{d-3}}{1+x^2}$$

$$r = r_0 - r_{0c} - A_d r^{\frac{d}{2}-1}$$

$d > 4$ | $r \sim (r_0 - r_{0c})$ | $\gamma = 1$

$2 < d < 4$ | $r \sim (r_0 - r_{0c})^{2/(d-2)}$ | $\gamma = \frac{2}{d-2}$

Homework

(1) Determine the steady state distribution for a 4 spin flip-and-exchange chain using periodic boundary conditions. Show that all interactions allowed by symmetry are generated provided the temperatures of the flip and the exchange bathes are different.

(2) Calculate the sysceptibility exponent in the spherical limit for the case when finite-temperature spin flips compete with random Levy exchanges (the Levy exponent of the range of exchanges is σ).

(3) Consider the one-dimensional driven particle system. Find a loop in the phase space which carries a probability current in the steady state and thus show that detailed balance is violated in this system.