Homework No.6 for Extreme Value Statistics **Deadline Apr. 24th, 22:00.**

(1) 100 pt.

Consider the extreme value distribution $P(z_N)$ of the largest $z_N = \max\{y_1, y_2, ..., y_N\}$ obtained from N independent draws in the $[0, \infty]$ interval using the parent distributions

$$P_0(y) = \frac{e^{-\sqrt{y}}}{2\sqrt{y}} \quad \text{(A)} \qquad , \qquad P_0(y) = \frac{3}{2}\sqrt{y}e^{-y^{3/2}} \quad \text{(B)}, \tag{1}$$

where A=Bendegúz and B=Lénárd. As one can easily show, the functions $P_0(y)$ are normalized in the $[0, \infty]$ interval.

We know that, for the given parent distributions (exponential decay), using appropriate shift and scaling, the limit distribution is the Gumbel distribution

$$\lim_{N\to\infty} P(z_N = a_N x + b_N) = P_G(x) = \frac{\pi}{\sqrt{6}} \exp\left[-\frac{\pi}{\sqrt{6}}x - \gamma_E - e^{-\frac{\pi}{\sqrt{6}}x - \gamma_E}\right] \quad . \tag{2}$$

where we used the standardization $\langle x \rangle = 0$, $\langle x^2 \rangle = 1$ for the limit distribution and γ_E is the Euler constant.

We also know that, for finite N, there are corrections to the above results:

$$P(z_N = a_N x + b_N) = P(x, N) \approx P_G(x) + q(N)\Phi(x) \quad . \tag{3}$$

where one expects that $q(N \to \infty) \to 0$ and $\Phi(x)$ is called the shape correction.

Tasks:

(i) Carry out simulations of finding z_N out of N draws from the parent distribution $P_0(y)$ and build the extreme value distributions for various N-s.

(ii) Use the standardization

$$\langle x \rangle = 0, \quad \langle x^2 \rangle = 1 \quad \to \quad x = \frac{z_N - \langle z_N \rangle}{\sigma}, \quad \sigma = \sqrt{\langle z_N^2 \rangle - \langle z_N \rangle^2} \tag{4}$$

to build the distribution function $P(x, N)$.

(iii) Analyse the simulation data by plotting

$$\frac{1}{q(N)} \left[P(x, N) - P_G(x)\right] \quad \text{vs.} \quad x \quad . \tag{5}$$

Find a functional form of $q(N)$ which results in a collapse of data for various N. Compare the resulting shape correction to the ones displayed in the lecture notes for the cases of exponential and Gaussian parents.