

Homework No.6 for Extreme Value Statistics**Deadline November 24th, 5PM.****(1)** 50 pt.

A simple quantity related to order statistics and used in astrophysics is the following ratio (introduced by Tremain and Richstone in 1977)

$$T_1 = \frac{\sigma_1}{d_{12}} \equiv \frac{\sqrt{\langle x_1^2 \rangle - \langle x_1 \rangle^2}}{\langle x_1 - x_2 \rangle} . \quad (1)$$

Here x_1 and x_2 are the largest and second largest luminosities of galaxies in a galaxy cluster and the averages are taken over galaxy clusters at a given distance (red shift). The value of $T_1 = \pi/\sqrt{6} = 1.28$ has been calculated for the Gumbel universality class (see lecture notes). The observations $T_1 \approx 0.4 \pm 0.2$, however, strongly disagree with the above number and the conclusion from this disagreement is that the galaxies cannot be considered as independent.

We know, however, that there are other limit distributions in extreme value statistics (EVS) of i.i.d. variables. Thus, it is of interest to apply them and, possibly, save the i.i.d. character of the luminosities. The task then is to consider T_1 for i.i.d. variables with parent distributions characterized by the parameter γ (see lecture notes). Study the cases $-\infty < \gamma \leq 1/2$ (note that the second moments of the limit distribution will have to be calculated and it does not exist for $\gamma \geq 1/2$) and show that $T_1(\gamma) > 1$ in the allowed range of γ -s (and conclude!).