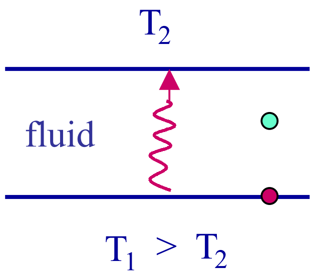
Nonequilibrium Dynamics and Fronts in Spin Chains

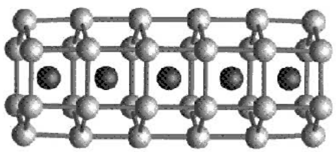
Questions: How to construct nonequilibrium steady states for quantum systems?
What are the general and distinct features of these steady states?
How do quantum systems relax to the steady state?



Aim: Understand first homogeneous steady states

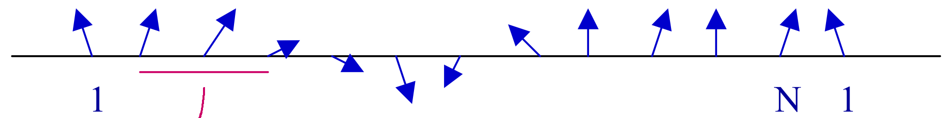
Results: Exact solutions (and numerical evaluations):

- transverse Ising model with j_E
- transverse XX model with j_E , j_M
  - steady state phase diagrams
  - correlations
  - probability distributions
  - relaxation -- scaling structure of fronts



Transverse Ising model with energy flux

$$\hat{H}_I = -\sum_{n=1}^{N} S_n^x S_{n+1}^x - \frac{h}{2}\sum_{n=1}^{N} S_n^z$$



local energy

$$\varepsilon_n = -\tfrac{1}{2} S_n^x (S_{n-1}^x + S_{n+1}^x) - h S_n^z$$

$$\dot{\varepsilon}_n = i[\hat{H}, \varepsilon_n] = j_{E,n-1\to n} - j_{E,n\to n+1}$$

local energy flux

Energy flux

$$\hat{J}_E = \frac{h}{4}\sum_{n=1}^{N} (S_n^x S_{n+1}^y - S_n^y S_{n+1}^x)$$

Find

$$\langle \psi \mid \hat{H}_I \mid \psi \rangle = \min$$

$$\langle \psi \mid \hat{J}_E \mid \psi \rangle = \mathrm{J}_E$$

Ground state:

$$\hat{H} = \hat{H}_I + \lambda \hat{J}_E$$

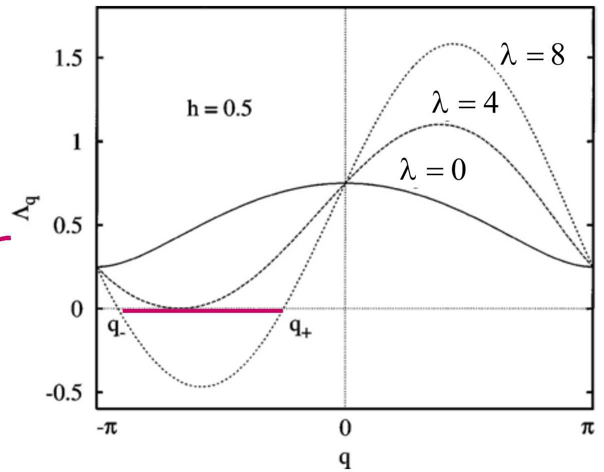
Nonequilibrium phase diagram

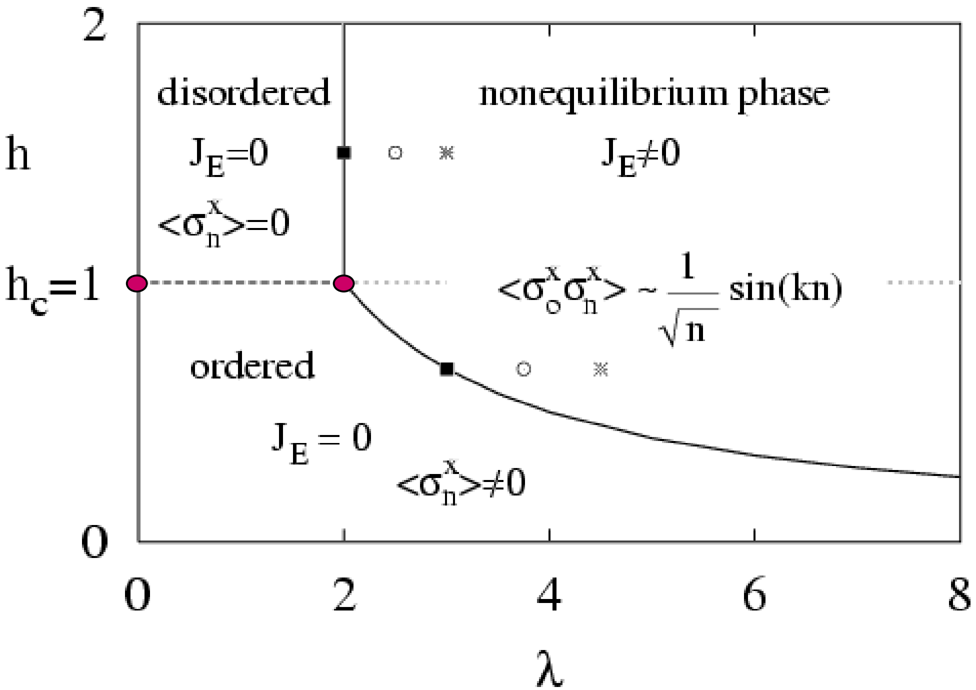
PRL **78**, 167 (1997)

$$\hat{H} = -\sum_{n=1}^{N} S_n^x S_{n+1}^x - \frac{h}{2}\sum_{n=1}^{N} S_n^z - \lambda \hat{J}_E$$

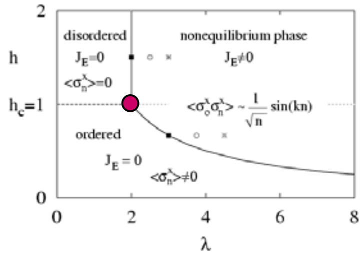
free fermions

$$E_N + \sum_q \Lambda_q c_q^+ c_q$$





● Power-law correlations:
Due to gapless excitation spectrum



Probability distributions (zero flux)

PRE **67**, 056129 (2003)

Nonordering field: $M_z = \sum S_n^z$

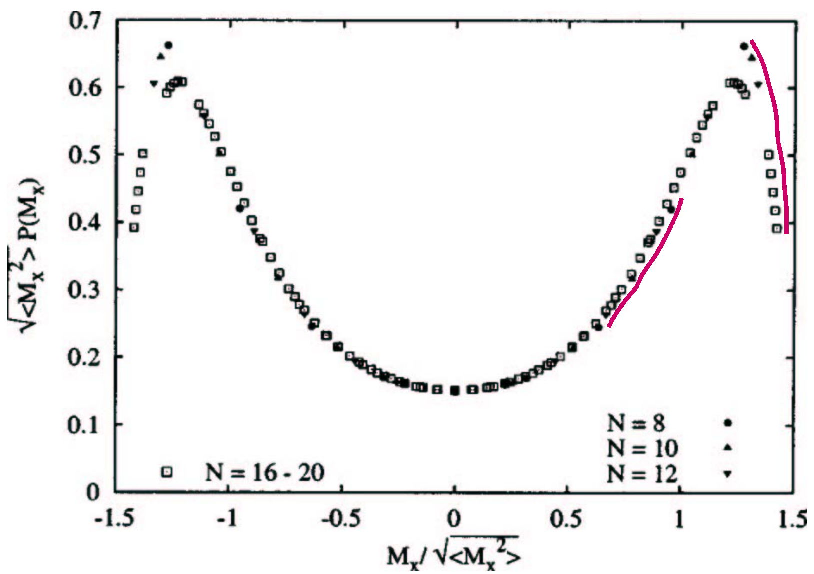
$P(M_z)$ is Gaussian even at criticality

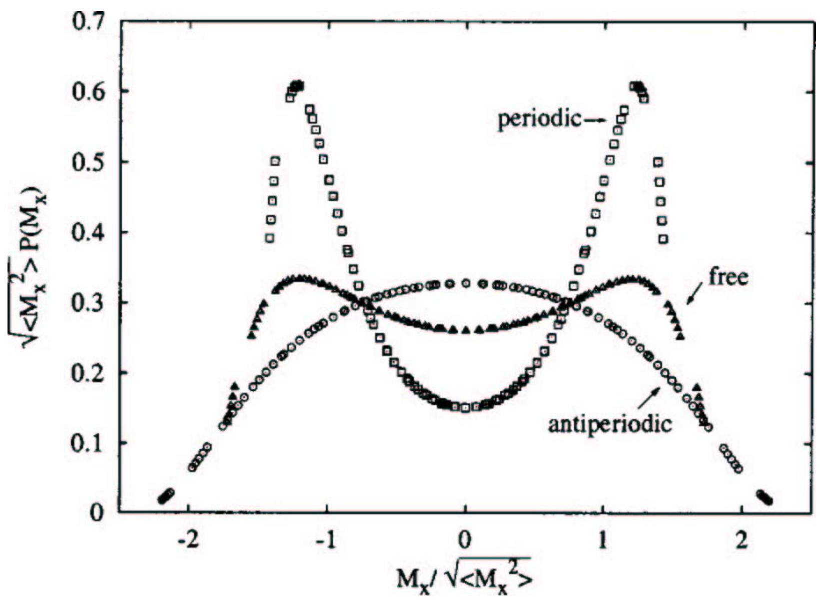
$$[\langle S_j^z S_{j+n}^z\rangle - \langle S_j^z\rangle\langle S_{j+n}^z\rangle]_{h=h_c} \propto 1/n^2$$

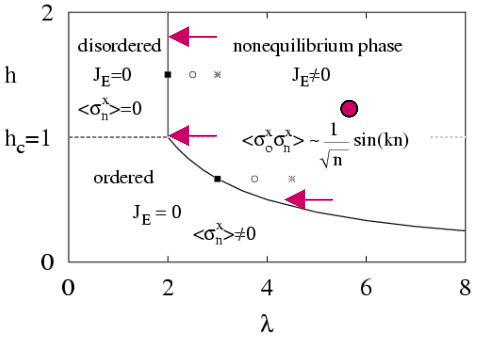
Ordering field: $M_x = \sum S_n^x$

Numerics:

- Boundary condition dependence of the scaling functions







Probability distributions (nonzero flux)

PRE **67**, 056129 (2003)

$$P(M_z) = \langle \delta(M_z - \sum S_n^z) \rangle$$

$$G(s) = \langle e^{-s\sum S_n^z} \rangle = e^{Ns/2} \langle e^{-s\sum_q c_q^+ c_q} \rangle$$

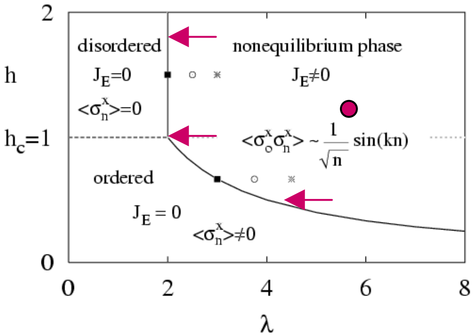
- $P(M_z)$ is Gaussian (exact)

Parameters of the Gaussian at small j_E

	$\langle M_z \rangle_{j_E} - \langle M_z \rangle_{j_E=0}$	$\langle (\delta M_z)^2 \rangle_{j_E} - \langle (\delta M_z)^2 \rangle_{j_E=0}$	j_E
$h>1$	$-\varepsilon^{1/2} \propto j_E$	$-\varepsilon^{1/2} \propto j_E$	$\varepsilon^{1/2}$
$h=1$	$-\varepsilon \propto j_E$	$-\varepsilon^{1/2} \propto j_E^{1/2}$	ε
$h<1$	$-\varepsilon^{3/2} \propto j_E^3$	$-\varepsilon^{1/2} \propto j_E$	$\varepsilon^{1/2}$

- Decrease of $\langle M_z \rangle$ and $\langle (\delta M_z)^2 \rangle$ is required for increasing j_E

$$\hat{j}_E \propto S_n^x S_{n+1}^y - S_n^y S_{n+1}^x$$



Probability distributions: Nonzero flux

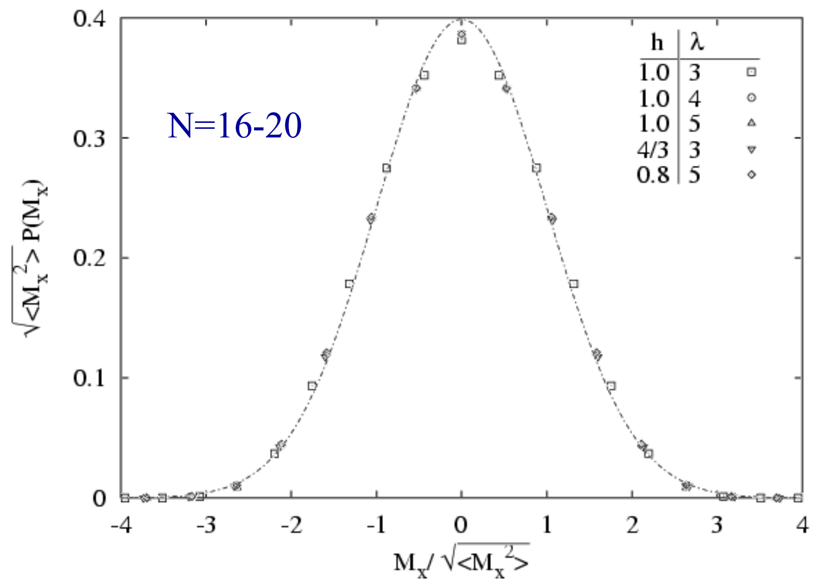
Ordering field: $P(M_x)$ is also a Gaussian (numerics)

In general: $j_E \Uparrow \;\rightarrow\; \langle M_x^2\rangle \Downarrow$

On h=1 line (h→1):

$$\langle S_j^x S_{j+n}^x\rangle \propto \frac{1}{n^{1/4}}\Phi(j_E^{1/2} n)$$

$$\langle M_x^2\rangle \propto j_E^{-3/8}$$



- Conclusion:

  Flux make the system stiffer

Transverse XX model with energy and magnetization flux

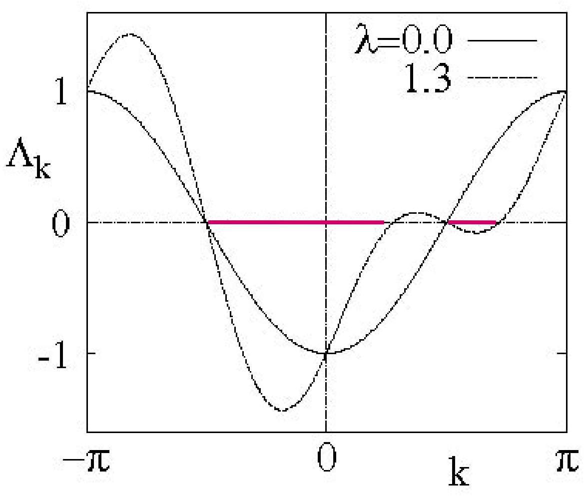
PRE **57**, 5187 (1998)

$$\hat{H}_{xx} = -\sum_{j=1}^{N-1}(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) - h\sum_{n=1}^{N} S_n^z$$

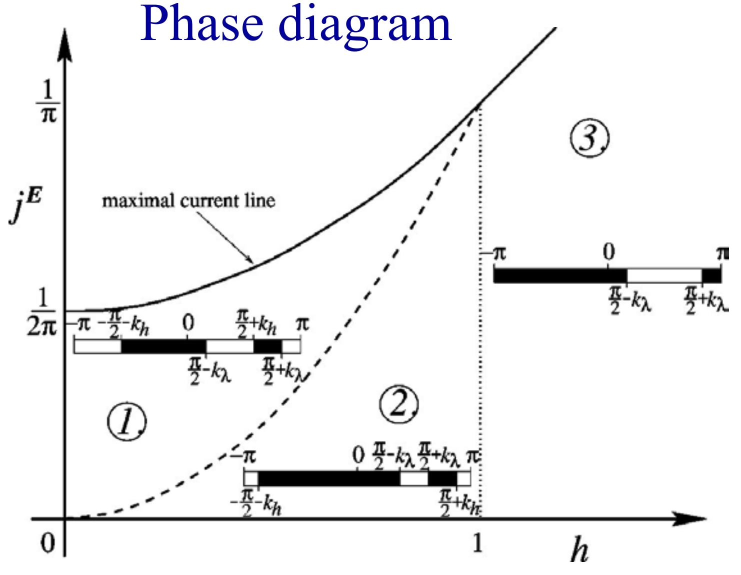
$$\hat{J}_E = \underbrace{\sum_{n=1}^{N} S_n^z (S_{n-1}^y S_{n+1}^x - S_{n-1}^x S_{n+1}^y)}_{\hat{J}_{E\,\mathrm{int}}} + \underbrace{h\sum_{n=1}^{N}(S_n^x S_{n+1}^y - S_n^y S_{n+1}^x)}_{-h\hat{J}_M}$$

$$\hat{H} = \hat{H}_{xx} + \lambda \hat{J}_E$$

Free fermion picture still works



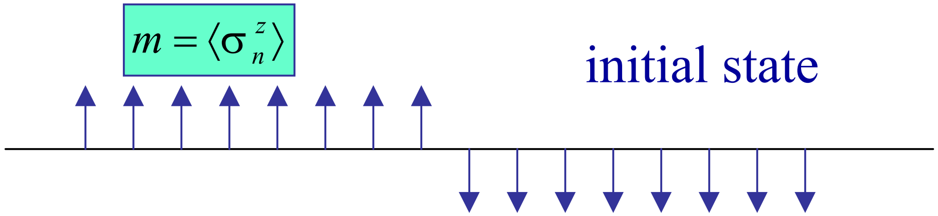
Phase diagram



Evolution from natural initial states

PRE **59**, 4912 (1999)

$$\hat{H}_{xx} = -\sum_{n=1}^{N-1} (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y)$$



initial state

Questions: Are there steady states in the $t \to \infty$ limit?

Can they be described by the $\langle \hat{H}_{xx} \rangle = \min \;;\; \langle \hat{J}_M \rangle = J_M$ approach?

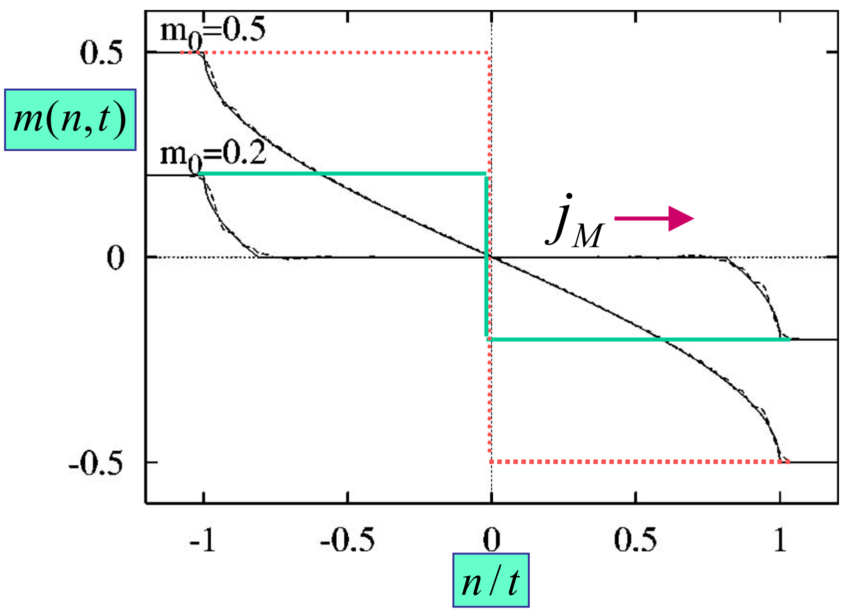
Time evolution (exact):

$m_0 = 1/2$

$$\langle S_n^z(t) \rangle = m(n,t) = -\tfrac{1}{2} \sum_{\ell=1-n}^{\ell=1+n} \mathrm{J}_\ell^2(t)$$

Scaling limit $\quad t \to \infty \quad \mathrm{n/t} = v$

$$m(n,t) \to -\tfrac{1}{\pi} \arcsin(\tfrac{n}{t})$$



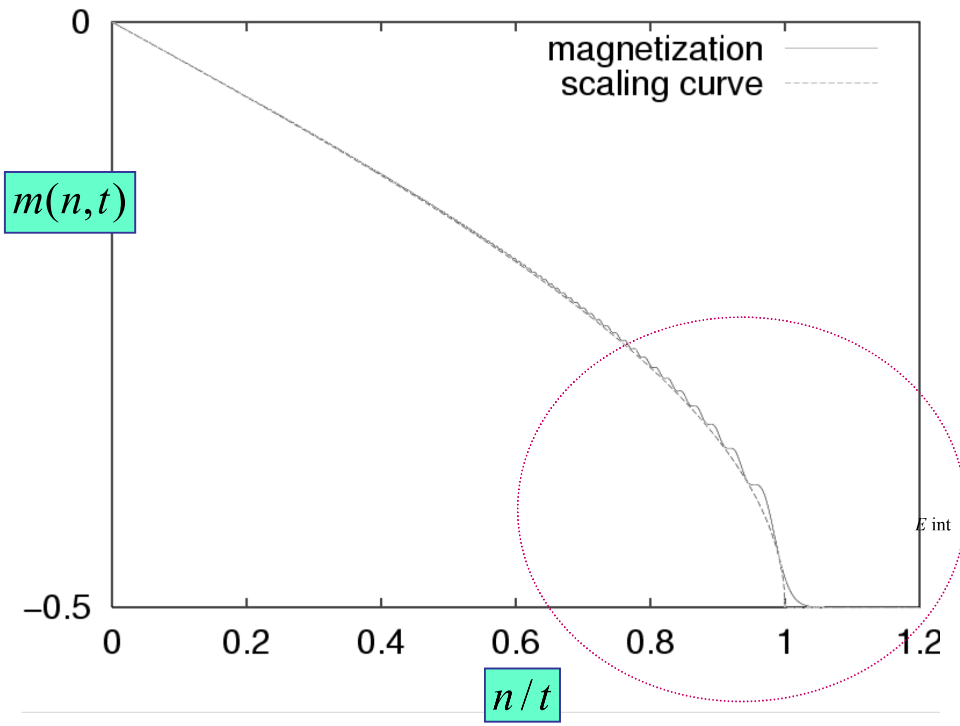
- J_M states are OK.

Problems in case of energy flux:

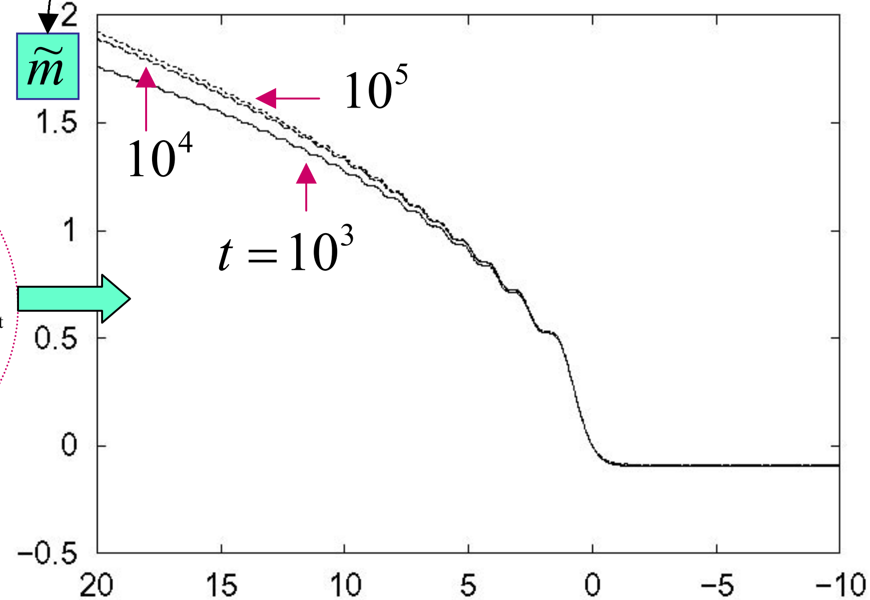
Y.Ogata,
PRE **66**, 016135 (2002)

Scaling structure of the front

V. Hunyadi, Z.R., L. Sasvári,
cond-mat/0312250



$$\tilde{m} = t^{1/3}[m(n,t) - m(t,t)]$$



$$z = (t-n)/t^{1/3}$$

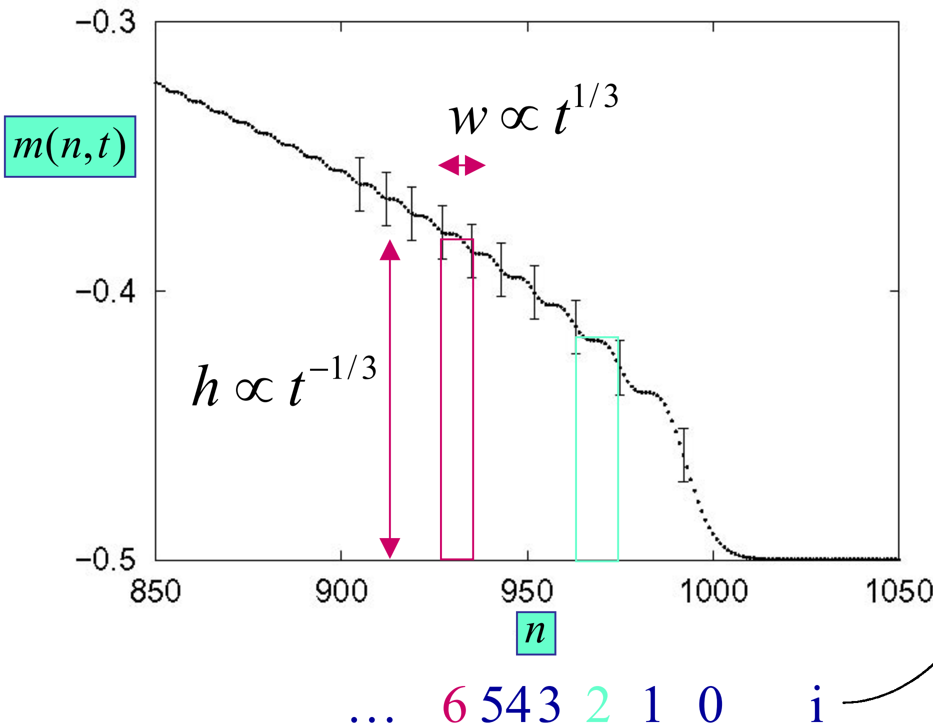
Scaling in the front:

$$m(n,t) - m(t,t) = \frac{1}{t^{1/3}} \Phi\left(\frac{t-n}{t^{1/3}}\right)$$

integral of the Airy function

- Front shape is formed early
- Width of the front $\propto t^{1/3}$

Quantized transport?



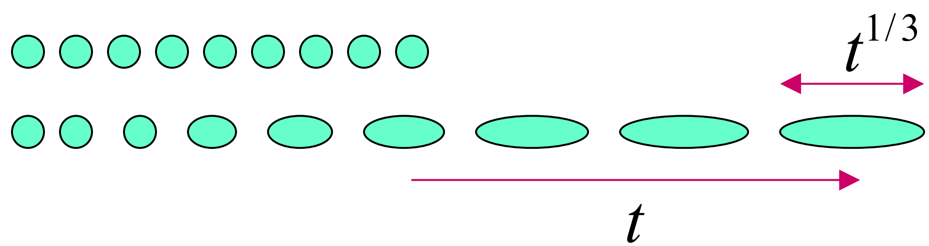
Areas under the steps are constant

$$hw \simeq 1$$

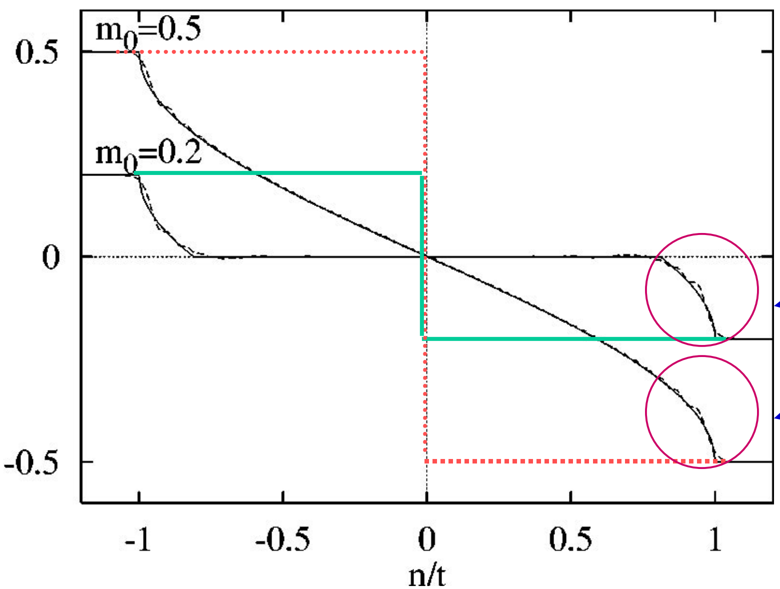
i	$t = 10^3$	10^5
1	1.018	1.038
2	0.971	1.009
3	1.037	0.993
.	.	.

- Steps carry one ↑ spin
- Classical picture

Subdiffusive spreading



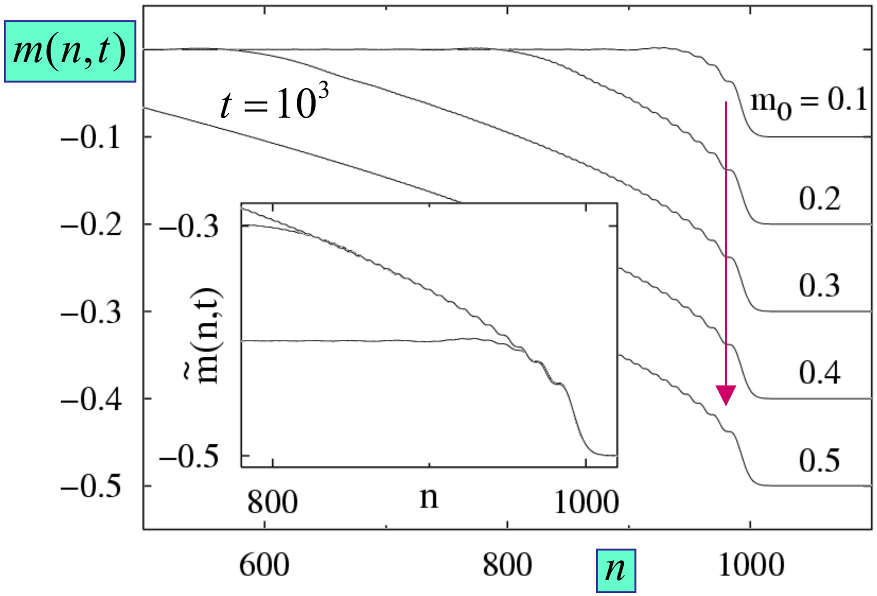
Macro-control of the number of steps

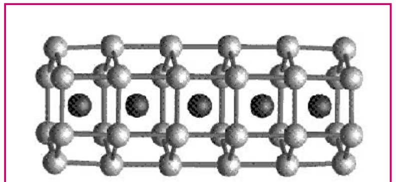


Practically identical profiles

m_0 determines the number of steps arriving at time t.

$$N(t, m_0) \approx \frac{\pi^2}{3} m_0^3 t$$





Bit manipulations in spin chains?

Problems: Finite T

Nonintegrability effects

Impurity effects